II a) $$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} k (x_2 - x_1)^2 + \frac{1}{2} k (x_3 - x_2)^2 + \frac{1}{2} (5k)(x_3 - x_1)^2 + \frac{1}{2} k x_3^2$$

$$L = T - V$$

equ. de Lagrange $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = F_i$

$$m_1 \ddot{x}_1 + k x_1 - k(x_2 - x_1) - (5k)(x_3 - x_1) = F_1$$

$$m_2 \ddot{x}_2 + k(x_2 - x_1) - k(x_3 - x_2) = F_2$$

$$m_3 \ddot{x}_3 + k(x_3 - x_2) + (5k)(x_3 - x_1) + k x_3 = F_3$$

$$\boxed{\begin{aligned} & m_1 \ddot{x}_1 + (7k) x_1 - k x_2 - (5k) x_3 = F_1 \\ & m_2 \ddot{x}_2 + (2k) x_2 - k x_1 - k x_3 = F_2 \\ & m_3 \ddot{x}_3 + (7k) x_3 - k x_2 - (5k) x_1 = F_3 \end{aligned}}$$

ok.

1b) $$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 (x_3 - x_2)^2$$

$$D = \frac{1}{2} \alpha_1 \dot{x}_1^2 + \frac{1}{2} \alpha_2 (\dot{x}_2 - \dot{x}_1)^2 + \frac{1}{2} \alpha_3 (\dot{x}_3 - \dot{x}_2)^2 + \frac{1}{2} \alpha_4 \dot{x}_2^2 + \frac{1}{2} \alpha_5 (\dot{x}_3 - \dot{x}_1)^2$$

equ. de Lagrange: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} = F_i$

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2 (x_2 - x_1) + \alpha_1 \dot{x}_1 - \alpha_2 (\dot{x}_2 - \dot{x}_1) - \alpha_5 (\dot{x}_3 - \dot{x}_1) = F_1$$

$$m_2 \ddot{x}_2 + k_2 (x_2 - x_1) - k_3 (x_3 - x_2) + \alpha_2 (\dot{x}_2 - \dot{x}_1) - \alpha_3 (\dot{x}_3 - \dot{x}_2) + \alpha_4 \dot{x}_2 = F_2$$

$$m_3 \ddot{x}_3 + k_3 (x_3 - x_2) + \alpha_3 (\dot{x}_3 - \dot{x}_2) + \alpha_5 (\dot{x}_3 - \dot{x}_1) = F_3$$

$$\boxed{\begin{aligned} & m_1 \ddot{x}_1 + (k_1 + k_2) x_1 + (\alpha_1 + \alpha_2 + \alpha_5) \dot{x}_1 - k_2 x_2 - \alpha_2 \dot{x}_2 - \alpha_5 \dot{x}_3 = F_1 \\ & m_2 \ddot{x}_2 + (k_2 + k_3) x_2 + (\alpha_2 + \alpha_3 + \alpha_4) \dot{x}_2 - k_2 x_1 - k_3 x_3 - \alpha_2 \dot{x}_1 - \alpha_3 \dot{x}_3 = F_2 \\ & m_3 \ddot{x}_3 + k_3 x_3 + (\alpha_3 + \alpha_5) \dot{x}_3 - k_3 x_2 - \alpha_3 \dot{x}_2 - \alpha_5 \dot{x}_1 = F_3 \end{aligned}}$$

ok.

II. a) $T = \frac{1}{2} m l^2 \dot{\phi}^2$

$V = mgl(1-\cos\phi)$

$D = \frac{1}{2} c a^2 \sin^2 \dot{\phi}$

$L = T - V = \frac{1}{2} m l^2 \dot{\phi}^2 + mgl\cos\phi - mgl$

b) equo, en incluent la dissipation.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} + \frac{\partial D}{\partial \dot{\phi}} = 0$$

$$ml^2\ddot{\phi} + mgl\sin\phi + ca^2 \sin\dot{\phi}\cos\dot{\phi} = 0$$

pt angles: $ml^2\ddot{\phi} + mgl\phi + ca^2\dot{\phi} = 0$

$$\ddot{\phi} + \frac{g}{l}\phi + \frac{ca^2}{ml^2}\dot{\phi} = 0$$

c) donc: $\omega_0^2 = \frac{g}{l}$

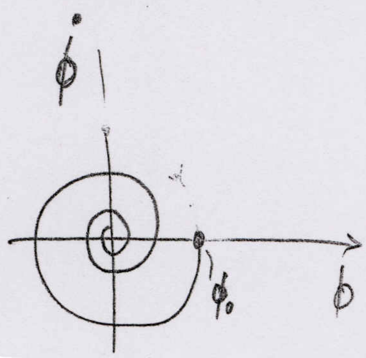
$2\lambda = \frac{ca^2}{ml^2}$ $\quad \lambda = \frac{ca^2}{2ml^2}$

ou $2\eta\omega_0 = \frac{ca^2}{ml^2}$ $\quad \eta = \frac{ca^2}{2ml^2}\frac{1}{\omega_0}$

d) amortissement critique pour $\eta = 1$,

$$\frac{ca_c^2}{2ml^2} = \omega_0 \quad , \quad a_c = \sqrt{\frac{2ml^2\omega_0}{c}} = l\sqrt{\frac{2m\omega_0}{c}}$$

e) pour $a < a_c$ on a : $\eta < 1$, sous-critique



f) $T = \frac{1}{2} m l^2 \dot{\phi}_1^2 + \frac{1}{2} m l^2 \dot{\phi}_2^2$

$$V = mgl\,(1-\cos\phi_1) + mgl\,(1-\cos\phi_2) + \frac{1}{2} k a^2 \underbrace{(\sin\varphi_1 - \sin\varphi_2)}_{\varphi_1,\varphi_2 \text{ pt} :\ \approx\ (\varphi_1-\varphi_2)}{}^2$$

$$L = T - V$$

équs. de Lagrange :

$$\left.\begin{aligned} m l^2 \ddot{\phi}_1 + mgl\,\phi_1 - k a^2(\varphi_1 - \varphi_2) &= 0 \\ m l^2 \ddot{\phi}_2 + mgl\,\phi_2 + k a^2(\varphi_1 - \varphi_2) &= 0 \end{aligned}\right\} \quad (1)$$

puls. propres :

$\phi_1 = \phi_1^{(0)} e^{i\omega t}$, $\phi_2 = \phi_2^{(0)} e^{i\omega t}$ dans (1) :

$$-m l^2 \omega^2 \phi_1^{(0)} + mgl\,\phi_1^{(0)} - k a^2 \phi_1^{(0)} + k a^2 \phi_2^{(0)} = 0$$
$$-m l^2 \omega^2 \phi_2^{(0)} + mgl\,\phi_2^{(0)} - k a^2 \phi_2^{(0)} + k a^2 \phi_1^{(0)} = 0$$

ou

$$\cancel{\begin{bmatrix} -m l^2 \omega^2 & mgl - k \\ mgl - k & -m l^2 \omega^2 \end{bmatrix} \begin{bmatrix} \phi_1^{(0)} \\ \phi_2^{(0)} \end{bmatrix} = 0}$$

$$\omega^2 \phi_1^{(0)} - \frac{g}{l}\phi_1^{(0)} + \frac{k a^2}{m l^2}\phi_1^{(0)} - \frac{k a^2}{m l^2}\phi_2^{(0)} = 0$$
$$\omega^2 \phi_2^{(0)} - \underbrace{\frac{g}{l}}_{\omega_0^2}\phi_2^{(0)} + \frac{k a^2}{m l^2}\phi_2^{(0)} - \underbrace{\frac{k a^2}{m l^2}}_{\kappa}\phi_1^{(0)} = 0$$

$$\underbrace{\begin{pmatrix} \omega^2 - \omega_0^2 + \kappa & -\kappa \\ -\kappa & \omega^2 - \omega_0^2 + \kappa \end{pmatrix}}_{\det = 0} \begin{pmatrix} \phi_1^{(0)} \\ \phi_2^{(0)} \end{pmatrix} = 0$$

$$(\omega^2-\omega_0^2+u)^2 - u^2 = 0$$

$$(\omega^2-\omega_0^2)^2 + 2(\omega^2-\omega_0^2)u^2 = 0$$

$$\omega^4 - 2\omega^2\omega_0^2 + \omega_0^4 + 2u^2\omega^2 - 2u^2\omega_0^2 = 0$$

$$\omega^4 - 2(\omega_0^2+u^2)\omega^2 + \omega_0^4 - 2u^2\omega_0^2 = 0$$

$$\omega_{1/2} = \frac{2(\omega_0^2+u^2) \pm \sqrt{4(\omega_0^2+u^2) - 4\cdot 1\,(\omega_0^4 - 2u^2\omega_0^2)}}{2}$$

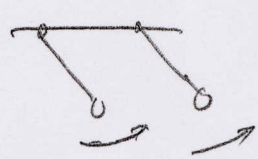
$$= (\omega_0^2+u^2) \pm \sqrt{(\omega_0^2+u^2)^2 - \omega_0^4 - 2u^2\omega_0^2}$$

$$= (\qquad) \pm \sqrt{\cancel{\omega_0^4} + \cancel{2\omega_0^2u^2} + u^4 - \cancel{\omega_0^4} - \cancel{2u^2\omega_0^2}}$$

$$= \omega_0^2 + u^2 \pm u^2$$

$$= \omega_0^2 + 2u^2 \quad \text{et} \quad \omega_0^2$$

$\omega_0^2 + 2u^2$



ω_0^2,

le ressort n'intervient pas