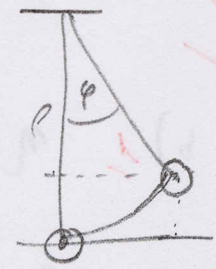
①

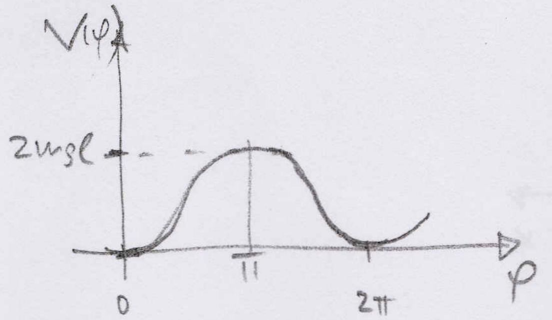
I. osc. non-linéaires : périodique, mais pas sinusoidal 1
pas de principe de superposition. 1

ex : pendule, van-der Pol 1 ③

II. a) 0.5 $V(\varphi) = mgh = mgl(1-\cos\varphi)$



0.5 $T = \frac{1}{2} m l^2 \dot{\varphi}^2$



①

b) $L = T - V = \frac{1}{2} m l^2 \dot{\varphi}^2 - mgl(1-\cos\varphi)$

equ de mouvement :

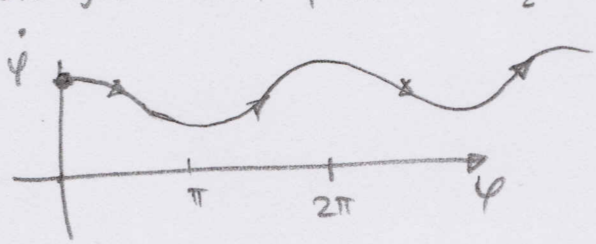
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0 \Rightarrow ml^2\ddot{\varphi} + mgl\sin\varphi = 0$$

$$\ddot{\varphi} + \frac{g}{l}\sin\varphi = 0$$

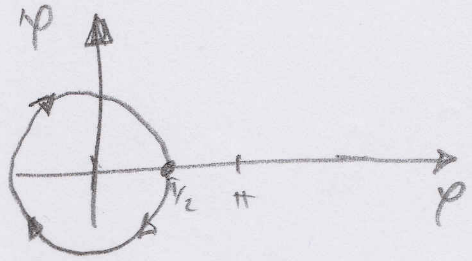
1 ①

c) energie cinétique : $T = \frac{1}{2} m l^2 \left(3\sqrt{g/l}\right)^2 = 4.5\, mgl > V(\pi)$

0.5 (i)



(ii)



0.5 ①

III. a) $|x_0| = \left|\frac{F}{m}\right| \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\eta^2\omega_0^2\omega^2}}$ (2)

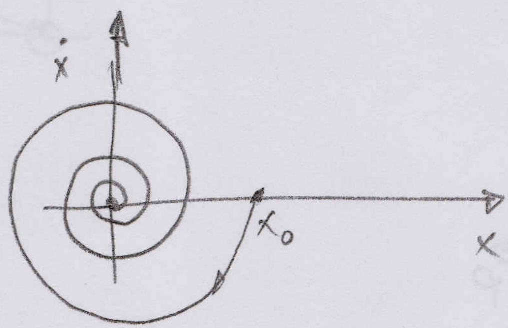
1 (1.5)

max pour: $\frac{d}{d\omega}|x_0| = 0 \Rightarrow \eta = \sqrt{\frac{\omega_0^2 - \omega^2}{\omega_0^2}}$

0,5 donc $\eta = \sqrt{1-(0.85)^2} \simeq 0.5$ sans dim.

b) 1 $\eta < 1$, donc amortissement sous-critique (1.5)

0,5



(3)

IV. a) dim = 2 (1)

b) $T = \frac{1}{2} M \dot{\xi}^2 + \frac{1}{2} J \dot{\vartheta}^2$

$= \frac{1}{2} M \dot{\xi}^2 + \frac{1}{4} M R^2 \dot{\vartheta}^2$ (1)

$V = \frac{1}{2} k \xi^2 + \frac{k}{2} (\xi - R\vartheta)^2 + \frac{(4k)}{2} \left(\frac{R}{2}\vartheta\right)^2 + \frac{k}{2} (R\vartheta)^2$

$= \frac{1}{2} k \xi^2 + \frac{k}{2} (\xi - R\vartheta)^2 + \frac{k}{2}\, 2 R^2 \vartheta^2$ (1)

$= \frac{1}{2} k \xi^2 + \frac{k}{2} \xi^2 - k R \xi \Theta + \frac{k}{2} R^2 \Theta^2 + k R^2 \Theta^2 = k \xi^2 + \frac{3k}{2} R^2 \Theta^2 - k R \xi \Theta$

c) $L = T - V$

eqn pour ξ :

$M \ddot{\xi} + k \xi + k (\xi - R\vartheta) = F$

$\ddot{\xi} + \frac{2k}{M} \xi - \frac{k}{M} R \vartheta = \frac{F}{M}$ (1)

$\frac{1}{2} M R^2 \ddot{\vartheta} - k R (\xi - R\vartheta) + k\, 2 R^2 \vartheta = 0$

$\frac{1}{2} M R^2 \ddot{\vartheta} + k R^2 \vartheta + 2 k R^2 \vartheta - k R \xi = 0$

$\ddot{\vartheta} + 6 \frac{k}{M} \vartheta - 2 \frac{k}{M} \frac{\xi}{R} = 0$ (1)

d) pulsations propres :

on pose : $\zeta = \zeta_0 e^{i\omega t}$, $\vartheta = \vartheta_0 e^{i\omega t}$

$$\underbrace{\begin{pmatrix} 2\omega_0^2-\omega^2 & -\omega_0^2 R \\ -2\omega_0^2 \frac{1}{R} & 6\omega_0^2-\omega^2 \end{pmatrix}}_{\det \overset{!}{=} 0} \begin{pmatrix} \zeta_0 \\ \vartheta_0 \end{pmatrix} = 0$$

$$(2\omega_0^2-\omega^2)(6\omega_0^2-\omega^2) - 2\omega_0^4 = 0$$

$$12\omega_0^4 - 2\omega_0^2\omega^2 - 6\omega_0^2\omega^2 + \omega^4 - 2\omega_0^4 = 0$$

$$\omega^4 - (8\omega_0^2)\omega^2 + 10\omega_0^4 = 0$$

$$\omega_{1/2}^2 = \frac{8}{2}\omega_0^2 \pm \frac{1}{2}\sqrt{64\omega_0^4 - 4\cdot 1\cdot 10\,\omega_0^4}$$

$$= 4\omega_0^2 \pm \omega_0^2\frac{1}{2}\sqrt{24}$$

$$= \omega_0^2(4\pm\sqrt{6})$$ 1

1

pour $\omega' = \omega_0^2(4+\sqrt{6})$ on a : $[2\omega_0^2 - (4+\sqrt{6})\omega_0^2]\zeta_1 = \omega_0^2 R\vartheta_0$

$$(2-4-\sqrt{6})\zeta_0 = \vartheta_0$$

$$-(2+\sqrt{6})\zeta_0 = \vartheta_0$$ 0,5 0,5

sens opposé

pour $\omega'' = 2\omega_0^2(2-\frac{1}{2}\sqrt{2})$ on a $[2\omega_0^2 - (4-\sqrt{6})\omega_0^2]\zeta_0 = \omega_0^2 R\vartheta_0$

$$(2-4+\sqrt{6})\zeta_0 = R\vartheta_0$$

$$(\sqrt{6}-2)\zeta_0 = R\vartheta_0$$ 0,5

même sens

0,5

e) i) $(2\omega_0^2 - \Omega^2)\zeta_0 - \omega_0^2 R \nu_0 = \dfrac{F_0}{M}$

ii) $(6\omega_0^2 - \Omega^2)\nu_0 - 2\omega_0 \dfrac{\zeta_0}{R} = 0$

avec ii) on a : $\zeta_0 = \dfrac{(6\omega_0^2 - \Omega^2) R \nu_0}{2\omega_0^2}$, dans i)

$$(2\omega_0^2 - \Omega^2)(6\omega_0^2 - \Omega^2)\frac{R\nu_0}{2\omega_0^2} - \omega_0^2 R \nu_0 = \frac{F_0}{M}$$

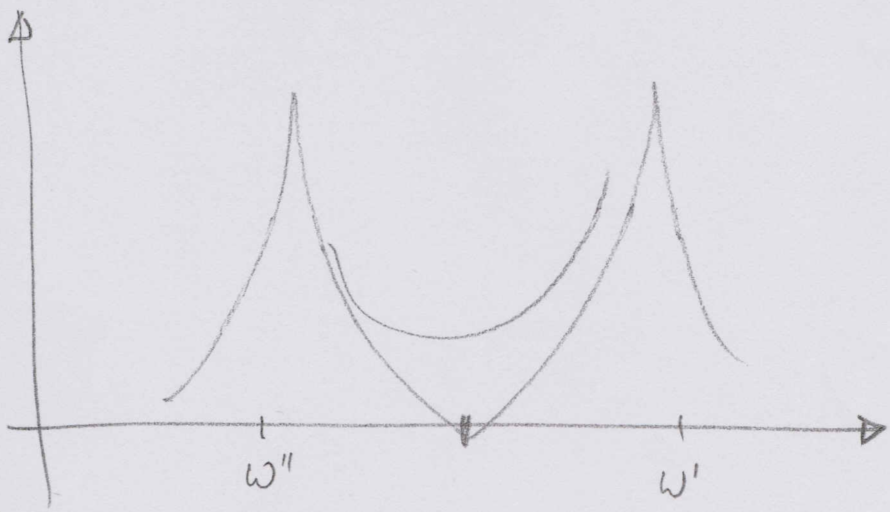
$$\left[(2\omega_0^2 - \Omega^2)(6\omega_0^2 - \Omega^2) - 2\omega_0^4\right] R\nu_0 = \frac{F_0}{M}(2\omega_0^2)$$

$$\nu_0 = \frac{1}{R}\frac{F_0}{M}\frac{2\omega_0^2}{\left[(2\omega_0^2 - \Omega^2)(6\omega_0^2 - \Omega^2) - 2\omega_0^4\right]} \quad 1$$

et $$\zeta_0 = \frac{F_0}{M}\frac{(6\omega_0^2 - \Omega^2)}{\left[(2\omega_0^2 - \Omega^2)(6\omega_0^2 - \Omega^2) - 2\omega_0^4\right]} \quad 1$$

2

f)



2/

$6\omega_0^2 - \Omega^2 = 0$

2