$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$ 0.5

0.5 $$V = \frac{1}{2} k (x_2 - x_1)^2 + \frac{1}{2} (3k)(x_3 - x_1)^2 + \frac{1}{2} k x_3^2$$ 0.5

~~0.5~~ $$L = T - V$$

equs de Lagrange : $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0$$ 0.5

$$\begin{cases} m_1 \ddot{x}_1 - k(x_2 - x_1) - (3k)(x_3 - x_1) = F_1 \\ m_2 \ddot{x}_2 + k(x_2 - x_1) = F_2 \\ m_3 \ddot{x}_3 + (3k)(x_3 - x_1) + k x_3 = F_3 \end{cases}$$

0.5 0.5 0.5

0.5 0.5 0.5

1

1 0.1 ~~4~~ 3

$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2$, $\quad J = \frac{1}{2} M R^2$

cond. de roulement sans glissement : $x = R\theta$, $\dot{x} = R\dot{\theta}$

$$= \frac{1}{2} M \dot{x}^2 + \frac{1}{4} M R^2 \dot{\theta}^2$$

$$= \frac{1}{2} M \dot{x}^2 + \frac{1}{4} M \dot{x}^2$$

(1) $$= \frac{1}{2} \left(\frac{3}{2} M\right) \dot{x}^2$$

(1) $V = 2 \cdot \frac{1}{2} k d^2$, $\quad d = a \sin\theta \approx a\theta$

$$= \frac{1}{2} (2k) \frac{a^2}{R^2} x^2$$

(0,5) $D = \frac{1}{2} c \dot{x}^2$

$L = T - V$, equ : $\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} + \frac{\partial D}{\partial \dot{x}} = 0}$ (0,5)

b) $\frac{3}{2} M \ddot{x} + (2k) \frac{a^2}{R^2} x + c\dot{x} = 0$

$$\ddot{x} + \frac{2c}{3M} \dot{x} + \frac{4k}{3M} \frac{a^2}{R^2} x = 0$$ (1)

c) on a donc : $\omega_0^2 = \frac{4k}{3M} \frac{a^2}{R^2}$ $\quad \omega_0 = \frac{a}{R} \sqrt{\frac{4k}{3M}}$ (0,5)

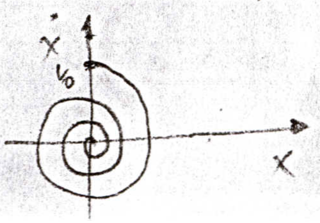
et $2\eta\omega_0 = \frac{2c}{3M}$ $\quad \eta = \frac{c}{3M} \cdot \frac{R}{a} \sqrt{\frac{3M}{4k}}$ (0,5)

$$= \frac{R}{2a} \frac{c}{\sqrt{3kM}}$$

d) point critique pour $\eta = 1$ donc

$$a_c = \frac{R}{2} \frac{c}{\sqrt{3kM}} \quad \text{ou} \quad a_c = \frac{c}{3M} \frac{1}{\omega_0}$$ (1)

e) pour $a > a_c$, on a $\eta < 1$, donc sous-critique (0,5)



(0,5)

$$T = \frac{1}{2} M \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 \quad (0.5)$$

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} k (x_2 - x_1)^2 \quad (0.5)$$

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0 \quad , \quad i = 1, 2$$

a)
$$\begin{cases} M\ddot{x}_1 + k x_1 - k(x_2 - x_1) = F \\ m\ddot{x}_2 + k(x_2 - x_1) = 0 \end{cases} \quad (0.5)$$

$$\begin{cases} \ddot{x}_1 + \frac{2k}{M} x_1 - \frac{k}{M} x_2 = F \\ \ddot{x}_2 + \frac{k}{m} x_2 - \frac{k}{m} x_1 = 0 \end{cases} = \begin{cases} \ddot{x}_1 + \omega_1^2 x_1 - \frac{k}{M} x_2 = F \\ \ddot{x}_2 + \omega_2^2 x_2 - \frac{k}{m} x_1 = 0 \end{cases}$$

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b) $F = 0$, $x_1 = X_1 e^{i\omega t}$, $x_2 = X_2 e^{i\omega t}$

$$\underbrace{\begin{pmatrix} \omega_1^2 - \omega^2 & -\frac{k}{M} \\ -\frac{k}{m} & \omega_2^2 - \omega^2 \end{pmatrix}}_{\det = 0} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = 0 \quad (0.5)$$

$$(\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2) - \frac{k^2}{mM} = 0$$

$$\omega^4 - (\omega_1^2 + \omega_2^2)\omega^2 + \omega_1^2\omega_2^2 - \frac{k^2}{mM} = 0 \qquad \text{avec : } \omega_1^2\omega_2^2 = \frac{2k^2}{mM}$$

$$\omega^4 - (\omega_1^2 + \omega_2^2)\omega^2 + \frac{1}{2}\omega_1^2\omega_2^2 = 0$$

$$\left.\begin{aligned} \omega'^2 &= \frac{1}{2}\left(\omega_1^2 + \omega_2^2 + \sqrt{\omega_1^4 + \omega_2^4}\right) \\ \omega''^2 &= \frac{1}{2}\left(\omega_1^2 + \omega_2^2 - \sqrt{\omega_1^4 + \omega_2^4}\right) \end{aligned}\right\} \quad (1)$$

c) I. $(\omega_1^2-\Omega^2)\,\underline{X}_1 - \frac{k}{M}\,\underline{X}_2 = \frac{F_0}{M}$

II. $(\omega_2-\Omega^2)\,\underline{X}_2 - \frac{k}{m}\,\underline{X}_1 = 0$

à partir de II. : $\underline{X}_1 = \frac{m}{k}(\omega_2^2-\Omega^2)\,\underline{X}_2$, dans I.

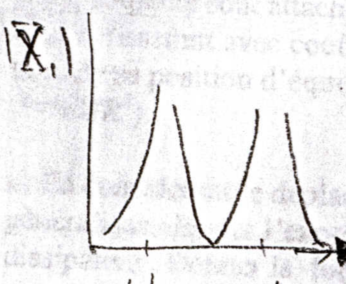
$$\left[\frac{m}{k}(\omega_1^2-\Omega^2)(\omega_2^2-\Omega^2) - \frac{k}{M}\right]\underline{X}_2 = \frac{F_0}{M}$$

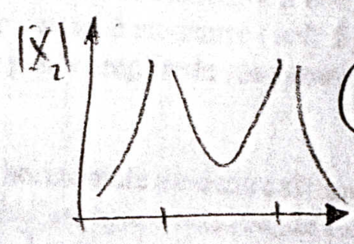
$$\left[(\omega_1^2-\Omega^2)(\omega_2^2-\Omega^2) - \frac{k^2}{mM}\right]\underline{X}_2 = \frac{k F_0}{mM}$$

0,5 $$\underline{X}_2 = \frac{k}{mM}\,\frac{F_0}{\left[(\omega_1^2-\Omega^2)(\omega_2^2-\Omega^2)-\frac{k^2}{mM}\right]}$$

0,5 $$\underline{X}_1 = \frac{F_0}{M}\,\frac{(\omega_2^2-\Omega^2)}{\left[(\omega_1^2-\Omega^2)(\omega_2^2-\Omega^2)-\frac{k^2}{mM}\right]}$$

d)





0,5

pour $\Omega = \omega', \omega''$ $\quad |X_1| \longrightarrow \infty$ $\quad$ résonance 0,5

$|X_2| \longrightarrow \infty$

pour $\Omega = \omega_2$ $\quad |X_1| = 0$ $\quad$ anti-résonance 0,5

$|X_2| = \left|\frac{F_0}{M}\right|$

m bouge de tel façon d'exercer une force sur M qui contrebalance la force externe 0,5