1 - $\hat{H}_{HF} = A\,\hat{I}.\hat{J}$ $\qquad \hat{F} = \hat{I} + \hat{J}$

Calcul dans la base "couplée" : états propres communs à (I^2, J^2, F^2, F_3).

$$\hat{I}\hat{J} = \frac{1}{2}\left[\hat{F}^2 - \hat{I}^2 - \hat{J}^2\right].$$

$$\Delta E_{HF} = \frac{A\hbar^2}{2}\left[F(F+1) - I(I+1) - J(J+1)\right].$$

2 - $n = 2$ $\begin{cases} \ell = 0 & j = \frac{1}{2} & 2s_{1/2} \\ \ell = 1 & j = \frac{1}{2}, \frac{3}{2} & 2p_{1/2} \quad 2p_{3/2} \end{cases}$

structure hyperfine : $|I - j| \leq F \leq I + j$

$2s_{1/2} \to F = \frac{1}{2}, \frac{3}{2}$ $\qquad A\hbar^2 = 81{,}7 \times \dfrac{1}{\frac{1}{2}\frac{3}{2} \times 8\,\frac{1}{2}} = \dfrac{81{,}7}{3} \simeq 27{,}23\,MHz$

pts $2p_{1/2} \to F = \frac{1}{2}, \frac{3}{2}$ $\qquad A\hbar^2 = 81{,}7 \times \dfrac{1}{\frac{1}{2}\frac{3}{2} \times 8 \times \frac{3}{2}} = \dfrac{81{,}7}{9} \simeq 9{,}077\,MHz$

$2p_{3/2} \to F = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ $\qquad A\hbar^2 = 81{,}7 \times \dfrac{1}{\frac{3}{2}\frac{5}{2} \times 8 \times \frac{3}{2}} = \dfrac{81{,}7}{45} \simeq 1{,}815\,MHz$

3 - Calcul de la correction énergétique associée à interaction hyperfine

$2s_{1/2}$ $\quad F = \frac{1}{2}$ $\qquad \Delta E_{HF} = \dfrac{27{,}23}{2}\left[\cancel{\tfrac{1}{2}\tfrac{3}{2}} - 2 - \cancel{\tfrac{1}{2}\tfrac{3}{2}}\right] = -27{,}23\,MHz.$

$F = \frac{3}{2}$ $\qquad \Delta E_{HF} = \dfrac{27{,}23}{2}\underbrace{\left[\tfrac{3}{2}\tfrac{5}{2} - 2 - \tfrac{1}{2}\tfrac{3}{2}\right]}_{1} = +\dfrac{27{,}23}{2} = 13{,}61$

$2p_{1/2}$ $\quad F = \frac{1}{2}$ $\qquad \Delta E_{HF} = \dfrac{9{,}077}{2}\left[\cancel{\tfrac{1}{2}\tfrac{3}{2}} - 2 - \cancel{\tfrac{1}{2}\tfrac{3}{2}}\right] = -9{,}077$

$F = \frac{3}{2}$ $\qquad \Delta E_{HF} = \dfrac{9{,}077}{2}\left[\;1\;\right] = \dfrac{9{,}077}{2} = 4{,}54$

$2p_{3/2}$ $\quad F = \frac{1}{2}$ $\qquad \Delta E_{HF} = \dfrac{1{,}815}{2}\left[\tfrac{1}{2}\tfrac{3}{2} - 2 - \tfrac{3}{2}\tfrac{5}{2}\right] = \dfrac{1{,}815}{2} \times (-5) = -4{,}54$

$F = 3/2$ $\qquad \Delta E_{HF} = \dfrac{1{,}815}{2}\left[\cancel{\tfrac{3}{2}\tfrac{5}{2}} - 2 - \cancel{\tfrac{3}{2}\tfrac{5}{2}}\right] = -1{,}815$

$2p_{3/2}$ $\quad F = \frac{5}{2} \quad \Delta E_{HF} = \frac{1,815}{2}\left[\frac{5}{2}\frac{7}{2} - 2 - \frac{3}{2}\frac{5}{2}\right] = 2,72$ MHz

4- Effet LAMB ⇒ structure hyperfine de $n = 2$

Structure fine = ?

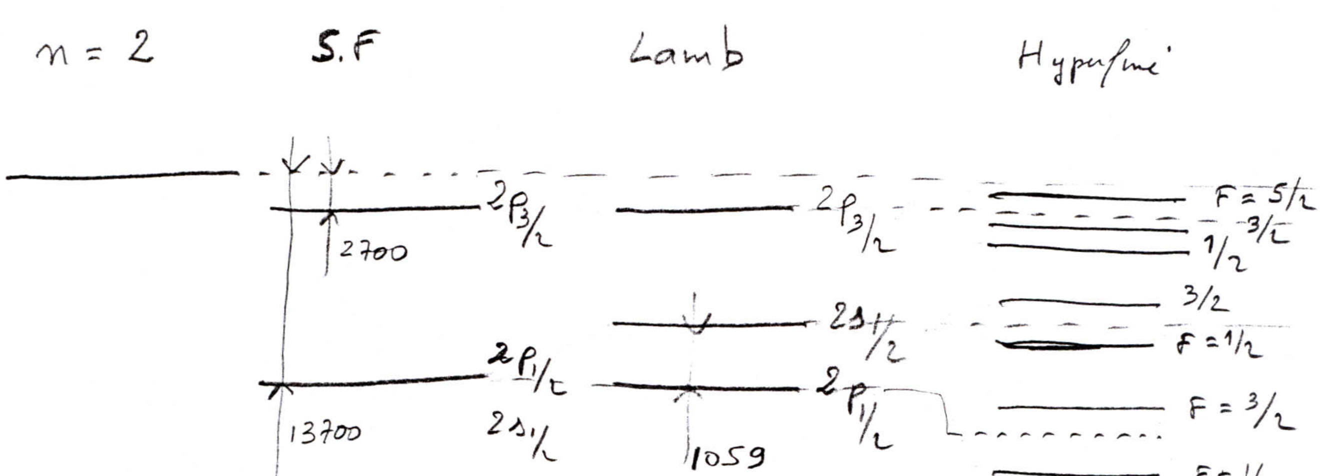
$$\Delta E_{SF} = \frac{+mc^2Z^4\alpha^4}{2n^3}\left[\frac{3}{4n} - \frac{2}{2j+1}\right].$$

pour $n = 2$ $\quad \frac{mc^2Z^4\alpha^4}{2n^3} = \frac{3,29.10^9 \times \left(\frac{1}{137}\right)^2 \times 1^2}{8} = 2,19\ 10^4$ MHz

$\boxed{1\ \text{pts}}$ $n = 2$ $\quad j = \frac{1}{2}$ $(2s_{1/2}, 2p_{1/2})$ $\quad \Delta E_{SF} = 2,19.10^4\left[\frac{3}{8} - \frac{2}{1+1}\right] = -1,37.10^4$

$j = \frac{3}{2}$ $(2p_{3/2})$ $\quad \Delta E_{SF} = 2,19.10^4\left[\frac{3}{8} - \frac{2}{3+1}\right] = -0,27.10^4$

d'où le diagramme :



Evaluation d'un rayon nucléaire

$^{13}_{7}N$ et $^{13}_{6}C$

1) Cinq termes
a_1 : Energie de volume
a_2 : Energie de surface
a_3 : Energie de répulsion électrostatique (protons)
a_4 : Energie d'asymétrie (N/Z)
a_5 : Energie d'appariement (effet quantique) parité de N/Z

2) $$dE_p = \frac{dq \times q}{4\pi\varepsilon_0 r} = \rho \frac{dV \times \rho V}{4\pi\varepsilon_0 r} = \frac{\rho^2}{4\pi\varepsilon_0 r}\left(\frac{4}{3}\pi r^3\right)\left(4\pi r^2 dr\right)$$

distribution de charges homogène

$$= \frac{4\pi\rho^2}{3\varepsilon_0} r^4 dr$$

$$E_p = \frac{4\pi\rho^2}{3\varepsilon_0}\int_0^R r^4 dr = \frac{4\pi\rho^2}{3\varepsilon_0}\frac{R^5}{5} \quad \text{avec } \rho = \frac{3Ze}{4\pi R^3} \text{ et } R = r_0 A^{1/3}.$$

$$\rightarrow E_p = \frac{3e^2}{5 \times 4\pi\varepsilon_0 r_0} Z^2 A^{-1/3} \qquad \Rightarrow a_3 = \frac{3e^2}{5 \times 4\pi\varepsilon_0 r_0}$$

3) Noyaux miroirs : nbre de neutrons et de protons échangés $\Rightarrow A$ constant
(A,Z) et $(A, A-Z)$

$$\Delta B = \underbrace{-a_3\left(A^2 - 2AZ\right)A^{-1/3}}_{\text{terme coulombien}} - a_4\left[\underbrace{(A-2(A-Z))^2 - (A-2Z)^2}_{\equiv 0}\right]A^{-1} + \frac{1}{2}\left[1+(-1)^A\right]A^{-3/4}\left((-1)^A(-1)^{-Z} - (-1)^Z\right)$$

A impair terme nul

$$A \text{ pair } \begin{cases} Z \text{ pair} \Rightarrow (-1)^A(-1)^{-Z} - (-1)^Z = 0 \\ Z \text{ impair} \Rightarrow (-1)^A(-1)^{-Z} - (-1)^Z = 0 \end{cases}$$

$\Rightarrow$ Tjs nul $\forall A$ pair ou impair

$$\Delta B_c = -a_3\left(A^2 - 2AZ\right)A^{-1/3} = B(A, A-Z) - B(A,Z)$$

Application $^{13}N / ^{13}C$

$$\underbrace{\underset{m_X}{^{13}_{7}N} \rightarrow \underset{m_Y}{^{13}_{6}C}}_{\text{noyaux nus}} + {}^{0}_{1}\beta^+ + \nu$$

$$\Delta B = B(\overset{A}{13}, \overset{Z}{6}) - B(13,7) = 13 a_3 \, 13^{-1/3}$$

4/

Energie au repos / liaison. $$m_X c^2 = \left[Z\,{}^1_1m + (A-Z)\,{}^1_0m \right] c^2 - B_X$$

énergies de liaison.

$$m_Y c^2 = \left[(A-Z)\,{}^1_1m + (A-(A-Z))\,{}^1_0m \right] c^2 - B_Y$$

$$\bullet\ (m_X - m_Y)c^2 = \left[(2Z-A)\,{}^1_1m + (A-2Z)\,{}^1_0m \right] c^2 + \underbrace{(B_Y - B_X)}_{(B_Y-B_X)_{\text{nucl}} + (B_Y-B_X)_C}$$

$$(B_Y-B_X)_{\text{nucl}} = 0 \quad \text{principe d'indépendance de charge.} \qquad (B_Y-B_X)_C = \Delta B_C$$

$$= \left[(2Z-A)\,{}^1_1m + (A-2Z)\,{}^1_0m \right] c^2 + a_3 (2AZ - A^2) A^{-1/3}$$

Application $\Longrightarrow$ $$(m_N - m_C)c^2 = \left[{}^1_1m - {}^1_0m \right] c^2 + 13\ 13^{-1/3} a_3$$

Cs de l'énergie β^+ $\bullet$ $$m_X c^2 = m_Y c^2 + m_e c^2 + T_{\beta max}$$

soit $$(m_X - m_Y)c^2 = m_e c^2 + T_{\beta max}.$$

ici $$(m_N - m_C)c^2 = m_e c^2 + T_{\beta max N}$$

Finalement : $$m_e c^2 + T_{\beta max N} = {}^1_1mc^2 - {}^1_0mc^2 + 13\ 13^{-1/3} a_3$$

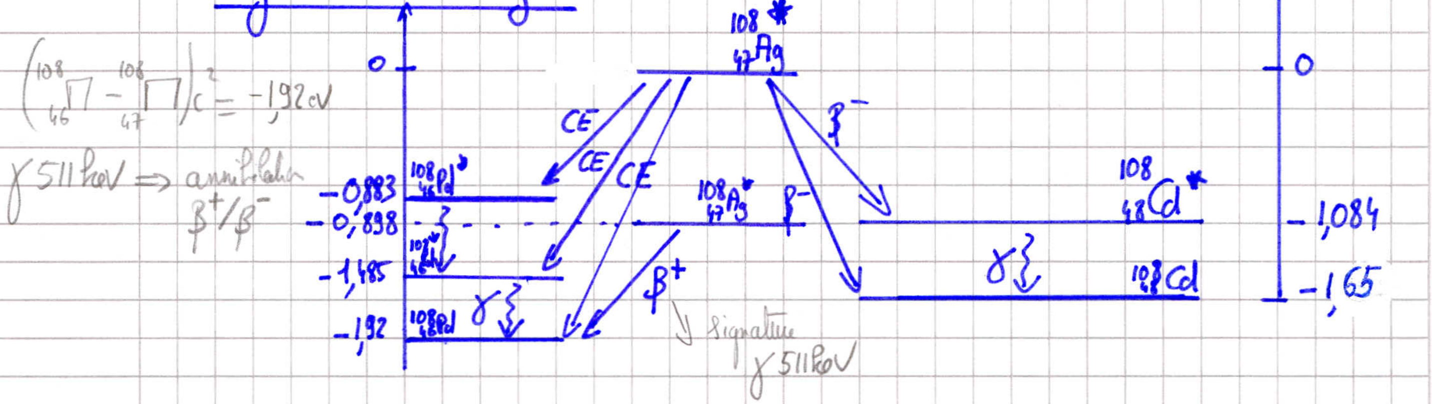
5/

soit $$a_3 = \frac{m_e c^2 + {}^1_0mc^2 - {}^1_1mc^2 + T_{\beta max N}}{13\ 13^{-1/3}}$$

AN : $a_3 = 0{,}4509$ MeV $\Longrightarrow$ $r_0 = 1{,}9$ fm

Rayonnement de l'^{108}Ag

$X \equiv CE$



Assemblage de quarks.

1) antiproton $\bar{u}\bar{u}\bar{d}$

2) antineutron $\bar{u}\bar{d}\bar{d}$