Étude de la désexcitation de l'état $2s$ de l'hydrogène

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a. $W_S = -\vec{D}.\vec{\mathcal{E}} = q\,z\,\mathcal{E} = q\,\mathcal{E}\,r\,\cos\theta$
$\langle n\ell, m_\ell|H_0|n\ell', m'_\ell\rangle = E_n\,\delta_{\ell,\ell'}\,\delta_{m_\ell,m'_\ell}$
$\langle n\ell, m_\ell|W_S|n\ell', m'_\ell\rangle \equiv q\,\mathcal{E}\,\langle R_{n,\ell}|r|R_{n,\ell'}\rangle\,\langle Y_\ell^{m_\ell}|\cos\theta|Y_{\ell'}^{m'_\ell}\rangle$
$\langle Y_0^0|\cos\theta|Y_0^0\rangle = 0$; $\quad\langle Y_1^{m_\ell}|\cos\theta|Y_1^{m'_\ell}\rangle = 0$
$\langle Y_0^0|\cos\theta|Y_1^{\pm1}\rangle = 0$; $\quad\langle Y_0^0|\cos\theta|Y_1^0\rangle = 1/\sqrt{3}$
$\langle R_{2,0}|r|R_{2,1}\rangle = -3\,a_0\,\sqrt{3}$
$\langle 2s|W_S|2s\rangle = 0$; $\quad\langle 2p|W_S|2p\rangle = 0$
$\langle 2s|W_S|2p, m_\ell = \pm1\rangle = 0$; $\quad\langle 2s|W_S|2p, m_\ell = 0\rangle = -3\,a_0\,q\,\mathcal{E}$ ....
$\hbar\omega_S = 3\,a_0\,q\,\mathcal{E} = 2,5\times10^{-27}\,\mathrm{J} = 1,6\times10^{-8}\,\mathrm{eV}$
$\omega_S = 2,5\times10^{-27}\,\mathrm{J}/1,054\times10^{-34}\,\mathrm{Js} = 2,4\times10^{7}\,\mathrm{s}^{-1}$ ....

b. 4 états. Les états $|2p, m_\ell = \pm1\rangle$ sont états propres dégénérés
de $H_0 + W_S$ avec valeur propre :
$\langle 2p, m_\ell = \pm1|(H_0 + W_S)|2p, m_\ell\rangle = E_2$
$E_2 = -Ry/4 = -5,45\times10^{-19}\,\mathrm{J} = -3{,}4\ \mathrm{eV}$. ....
Les deux autres états propres résultent de la diagonalisation de la matrice $H_0 + W_S$ dans le sous-espace 2x2 des états $|2s\rangle$ et $|2p, m_\ell = 0\rangle$:
$|\psi_+\rangle = (1/\sqrt{2})\,[|2s\rangle + |2p, m_\ell = 0\rangle]$; $\quad E_+ = E_2 - \hbar\omega_S$ ....
$|\psi_-\rangle = \left[1/\sqrt{2})\,(|2s\rangle - |2p, m_\ell = 0\rangle\right]$; $\quad E_- = E_2 + \hbar\omega_S$ ....

c. $\Gamma_+ = \Gamma_- = \Gamma_{2p}/2 = 3,1\times10^{8}\,\mathrm{s}^{-1}$. ....
Les deux autres états non perturbés
se désexcitent avec un taux $\Gamma_{2p} = 6,2\times10^{8}\,\mathrm{s}^{-1}$. ....
Tous les états seront désexcités pendant la durée
de l'expérience ($T_{exp} \simeq \times10^{-5}$ s). ....

Effet Zeeman

1- $E = -\vec{\mu}\,\vec{B}$

avec $\vec{\mu} = \sum \mu_i = \vec{\mu}_L + \vec{\mu}_S + \vec{\mu}_I$

$\vec{\mu}_L = -g_L \mu_B / \hbar \; \vec{L}$ avec $g_L \equiv 1$

$\vec{\mu}_S = -g_S \mu_B / \hbar \; \vec{S}$ avec $g_S \simeq 2$

$\vec{\mu}_I = g_I \frac{\mu_N}{\hbar} \vec{I}$ avec g_I (proton) $\simeq 5,58$

$\Rightarrow \hat{H}_Z = -\left[(\vec{L} + 2\vec{S}) \frac{\mu_B}{\hbar} + g_I \frac{\mu_N}{\hbar} \vec{I} \right] . \vec{B}$

$\vec{B} \,//\, Oz \Rightarrow \quad \hat{H}_Z = (\hat{L}_z + 2\hat{S}_z) \frac{\mu_B}{\hbar} B_0 - g_I \frac{\mu_N}{\hbar} B_0$

$\frac{\mu_N}{\mu_B} \sim \frac{m_e}{m_N} \leq \frac{1}{1836} \Rightarrow \hat{H}_Z \simeq (\hat{L}_z + 2\hat{S}_z) \frac{\mu_B}{\hbar} B_0$

2- a si B assez grand action sur $|n, l, s, m_l, m_s\rangle$.

$\Rightarrow \Delta E_Z = \mu_B B_0 (m_l + 2 m_s)$

$n = 1 \Rightarrow l = 0 \Rightarrow m_l = 0$ et $m_s = \pm \frac{1}{2}$

$n = 1 \quad \Delta E_Z = \pm \mu_B B_0.$

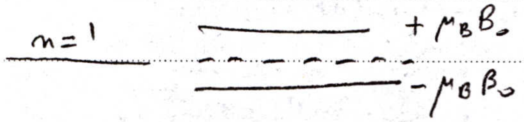
$n = 2 \nearrow l = 0 \quad m_l = 0 \quad m_s = \pm \frac{1}{2}$

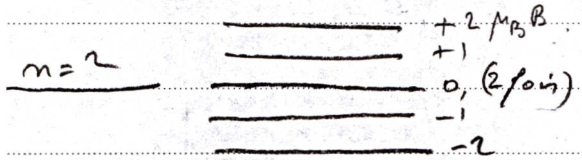
$n = 2 \searrow l = 1 \quad m_l = 0, \pm 1 \quad m_s = \pm \frac{1}{2}$

$\ell = 0 \qquad \Delta E_Z = \pm \mu_B B_0$

$n = 2$

$$\ell = 1 \begin{cases} m_\ell = 0 & \Delta E_Z = \pm \mu_B B_0 \\ m_\ell = -1 & \Delta E_Z = (-1 \pm 1)\mu_B B = 0, -2\mu_B B \\ m_\ell = +1 & \Delta E_Z = (1 \pm 1)\mu_B B = 0, +2\mu_B B \end{cases}$$





b- $h\nu = E_n - E_{n'}$ avec $E_n = E_{n_0} + (m_\ell + 2m_s)\mu_B B$

$$h\nu = \left[E_{n_0} + (m_\ell + 2m_s)\mu_B B\right] - \left[E_{n'_0} + (m'_\ell + 2m'_s)\mu_B B\right]$$

$\Delta \ell = \pm 1$, $\Delta m_\ell = 0, \pm 1$, $\Delta m_s = 0 \Rightarrow$

$$h\nu = \left(E_{n_0} - E_{n'_0}\right) + \text{``}\Delta m_\ell\text{''}\mu_B B,$$

$$h\nu = \left(E_{n_0} - E_{n'_0}\right) + \begin{matrix} \mu_B B \\ 0 \\ -\mu_B B \end{matrix}$$

c- $|\Delta(h\nu)| = \mu_B B = 9{,}27.10^{-24} \times 0{,}5 = 4{,}635.10^{-24}\ \mathrm{J}$

$h\nu = E = \dfrac{hc}{\lambda} \Rightarrow$ H: $n = 2 \rightarrow n = 1$ $\quad h\nu = E_2 - E_1$

$\lambda = \dfrac{hc}{E}$

$h\nu = -\dfrac{13{,}6}{4} - (-13{,}6) = 10{,}2\ \mathrm{eV}$

$= 1{,}632.10^{-18}\ \mathrm{J}$

$$|\Delta\lambda| = \frac{hc}{E^2}\Delta E = \frac{6.62.10^{-34}.3.10^{8}}{(1{,}632.10^{-18})^2}.4{,}635.10^{-24} = 3{,}45.10^{-13}\ \mathrm{m}$$

$= 3{,}45.10^{-3}\ \text{Å}$

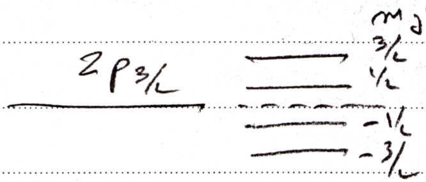
$3 - H_Z = \frac{\mu_B}{\hbar} g_J J_z B$

a) $\Delta E_Z = g_j \mu_B B M_J$

b) $n = 2,\ \ell = 1 \Rightarrow j = \frac{1}{2}, \frac{3}{2}$

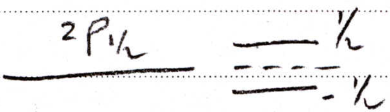
$|2, 1, \frac{1}{2}, \frac{1}{2}, \pm\frac{1}{2}\rangle \qquad 2P_{1/2}$

$$\left.\begin{cases} |2, 1, \frac{1}{2}, \frac{3}{2}, \pm\frac{1}{2}\rangle \\ |2, 1, \frac{1}{2}, \frac{3}{2}, \pm\frac{3}{2}\rangle \end{cases}\right\} 2P_{3/2}$$



$\pm \Delta E = g_j \mu_B B m_j$

avec $g_j = 1 + \frac{\frac{3}{2}\frac{5}{2} + \frac{1}{2}\frac{3}{2} - 1.2}{2\frac{3}{2}(\frac{5}{2})} = 1 + \frac{\frac{15}{4} + \frac{3}{4} - \frac{8}{4}}{\frac{30}{4}} = \frac{4}{3}.$



avec $g_j = 1 + \frac{\frac{1}{2}\frac{3}{2} + \frac{1}{2}\frac{3}{2} - 1.2}{2\frac{1}{2}\frac{3}{2}} = 1 + \frac{\frac{6}{4} - \frac{8}{4}}{\frac{6}{4}} = 1 - \frac{2}{6} = \frac{2}{3}$

Fission de l'uranium 235 en noyaux de krypton et de baryum

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{90}_{36}Kr + ^{142}_{Z}Ba + a\,^{1}_{0}n$$

1) Détermination de Z et de a

$$Z = 92 - 36 = 56$$

$$a = (235+1) - (90+142) = 4.$$

2) Energie libérée $\quad Q = \sum_i m_i c^2 - \sum_f m_f c^2$

$$= {}^{235}_{92}m\,c^2 + m_n c^2 - {}^{90}_{36}m\,c^2 - {}^{142}_{56}m\,c^2 - 4 m_n c^2$$

AN: $Q = 0{,}18185 \times 931{,}48 = 169{,}39\ MeV$

3) Energie libérée par 1 kg de ^{235}U

$$Q = \frac{169{,}39\ 10^{6} \times 1{,}602\ 10^{-19}}{235 \times 1{,}67\ 10^{-27}} \sim 6{,}9\ 10^{13}\ J$$

Durée de consommation $\quad \Delta t = \frac{Q}{P} \longrightarrow \Delta t = \frac{6{,}9\ 10^{13}}{500\ 10^{6}} = 1{,}38\ 10^{5}\ s$

$P = 500\ MW$ $\qquad\qquad = 1{,}6\ j$

4) $^{90}_{36}Kr \cdots\cdots\to {}^{90}_{40}Zr$ $\qquad$ pas $\left|\begin{array}{l}\alpha \text{ car } Z \nearrow \\ \beta^+ \\ CE\end{array}\right| \Rightarrow \beta^-$ car A cst

$$^{90}_{36}Kr \longrightarrow {}^{90}_{37}Rb + \beta^- + \bar{\nu}$$

$$\hookrightarrow {}^{90}_{38}Sr + \beta^- + \bar{\nu}$$

$$\hookrightarrow {}^{90}_{39}Y + \beta^- + \bar{\nu}$$

$$\hookrightarrow {}^{90}_{40}Zr + \beta^- + \bar{\nu}$$

Radiothérapie

1) $^{60}_{27}Co \longrightarrow {}^{60}_{28}Ni$

$A = \lambda N$

$\Leftrightarrow N = \dfrac{A}{\lambda}$

calcul de λ $\quad \lambda = \dfrac{\ln 2}{T} = 4,168\ 10^{-9}\ s^{-1} \Rightarrow N = 5,327\ 10^{22}$ noyaux

2) ^{99}Tc traceur radioactif

$^{99}Mo \longrightarrow {}^{99}Tc$
$T_{1/2} = 67h$.

$^{99}Tc^{*} \xrightarrow[\gamma]{} {}^{99}Tc$
$T_{1/2} = 6h$

Détection du photon γ

a) $^{99}_{42}Mo \longrightarrow {}^{99}_{43}Tc^{*} + {}^{0}_{-1}\beta^{-} + \bar{\nu}$

b) $A = 8,2\ 10^{7}\ Bq \Longrightarrow N_\gamma = 8,2\ 10^{7}\ s^{-1}$

c) $A = \lambda N = 38\ Bq$ $\quad \lambda = \dfrac{\ln 2}{T}$ $\quad T = 6 \times 3600$

$\lambda = 3,209\ 10^{-5}\ s^{-1}$

soit $N = \dfrac{A}{\lambda} = 1,184\ 10^{6}$ noyaux excités