M 1. Sci Phys — Mars 2006

(1) $n = 3$.

$|\, n \quad \ell \quad s \quad j \quad m_j \,\rangle$

18 états

n	ℓ	s	j	m_j	
3	0	$\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	$3s_{1/2}$
3	1	$\frac{1}{2}$	$\frac{1}{2}$	$\pm\frac{1}{2}$	$3p_{1/2}$
3	1	$\frac{1}{2}$	$\frac{3}{2}$	$\pm\frac{3}{2}$	$3p_{3/2}$
"	"	"	"	$\pm\frac{1}{2}$	
3	2	$\frac{1}{2}$	$\frac{3}{2}$	$\pm\frac{3}{2}$	$3d_{3/2}$
"	"	"	"	$\pm\frac{1}{2}$	
3	2	$\frac{1}{2}$	$\frac{5}{2}$	$\pm 5/2$	$3d_{5/2}$
"	"	"	"	$\pm 3/2$	
"	"	"	"	$\pm\frac{1}{2}$	

(2) a) niveaux différents $(3s_{1/2}, 3p_{1/2})$

$(3p_{3/2}, 3d_{3/2})$

$(3d_{5/2})$

$\Delta E_{n,j}$: $\boxed{E_{3,1/2}}$, $\boxed{E_{3,3/2}}$, $\boxed{E_{3,5/2}}$ — 3 niveaux différents

b) $Z = 1$, $mc^2\alpha^4 \simeq 1{,}45 \cdot 10^{-3}\, eV$; $n = 3$ $\left(\frac{-13{,}6}{9}\right)$

$+\frac{mc^2 Z^4 \alpha^4}{2n^3} \simeq +2{,}685 \cdot 10^{-5}\, eV$ $\cancel{12{,}5}$

$\Delta E_{3,\frac{1}{2}} = +2{,}685 \cdot 10^{-5}\left[\frac{3}{12} - \frac{2}{1+1}\right] \simeq -2{,}01 \cdot 10^{-5}\, eV$

$\Delta E_{3,\frac{3}{2}} = 2{,}685 \cdot 10^{-5}\left[\frac{3}{12} - \frac{2}{3+1}\right] \simeq -0{,}671 \cdot 10^{-5}\, eV$

$\Delta E_{3,\frac{5}{2}} = 2{,}685 \cdot 10^{-5}\left[\frac{3}{12} - \frac{2}{5+1}\right] \simeq -0{,}223 \cdot 10^{-5}\, eV$

3

a) $\hat{H}_Z |n,\ell,s,j,m_j\rangle \equiv g_j \frac{\mu_B}{\hbar} B \hat{J}_3 |n,\ell,s,j,m_j\rangle$

$$\Delta E_Z = \langle n,\ell,s,j,m_j | \hat{H}_Z | n,\ell,s,j,m_j\rangle$$

$$\boxed{\Delta E_Z = g_j \mu_B B m_j}$$

b) pour $3d_{5/2}$, $m_j = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}$

$j = \frac{5}{2}$, $\ell = 2$ $\left(s = \frac{1}{2}\right)$ $\Rightarrow g_j = 1 + \dfrac{\frac{5}{2}\cdot\frac{7}{2} + \frac{1}{2}\cdot\frac{3}{2} - (2\cdot 3)}{5\left(\frac{7}{2}\right)}$

$$g_j = 1 + \frac{\frac{35}{4} + \frac{3}{4} - \frac{24}{4}}{\frac{70}{4}} = 1 + \frac{14}{70} = 1 + \frac{1}{5}$$

$g_j = \frac{6}{5}$. ; $\underline{\mu_B B \simeq 5,8.10^{-5}\,eV}$

pour 1 Tesla

$$\begin{cases} \Delta E_Z = \pm 3,48.10^{-5}\,eV & (m_j = \pm\frac{1}{2}) \\ \Delta E_Z = \pm 1,04.10^{-4}\,eV & (m_j = \pm\frac{3}{2}) \\ \Delta E_Z = \pm 1,74.10^{-4}\,eV & (m_j = \pm\frac{5}{2}) \end{cases}$$

$1,74 \times 1,6$

1044
174
2784

10^{-23}

Correction du même ordre de grandeur ou plus grande que la correction de structure fine ! Contraire aux hypothèses.

c) Avec $\Delta E_{SF} \simeq 0,5.10^{-5}\,eV$

$\Rightarrow \mu_B B \lesssim 0,5.10^{-6}\,eV$

soit $B \lesssim \dfrac{0,5.10^{-6}}{5,8.10^{-5}} \simeq 0,0086\,T$

Energie de liaison de différents noyaux

1) $^{4}_{2}\alpha$ $\qquad \mathcal{E}_\ell = 2 m_p c^2 + 2 m_n c^2 - m_\alpha c^2$

AN: $\mathcal{E}_\ell = 2 \times 938{,}27 + 2 \times 939{,}565 - 3727{,}27 = 28{,}4 \text{ MeV}$

$\dfrac{\mathcal{E}_\ell}{A} = 7{,}1 \text{ MeV}$

2) ^{238}U $\qquad \mathcal{E}_\ell = 92\, m_p c^2 + 146\, m_n c^2 - m_U c^2$

AN: $\mathcal{E}_\ell = 92 \times 938{,}27 + 146 \times 939{,}565 - 221697{,}7 = 1799{,}6 \text{ MeV}$

$\dfrac{\mathcal{E}_\ell}{A} = 7{,}6 \text{ MeV}$

Désintégrations nucléaires et fonctionnement du corps humain

□ Potassium naturel

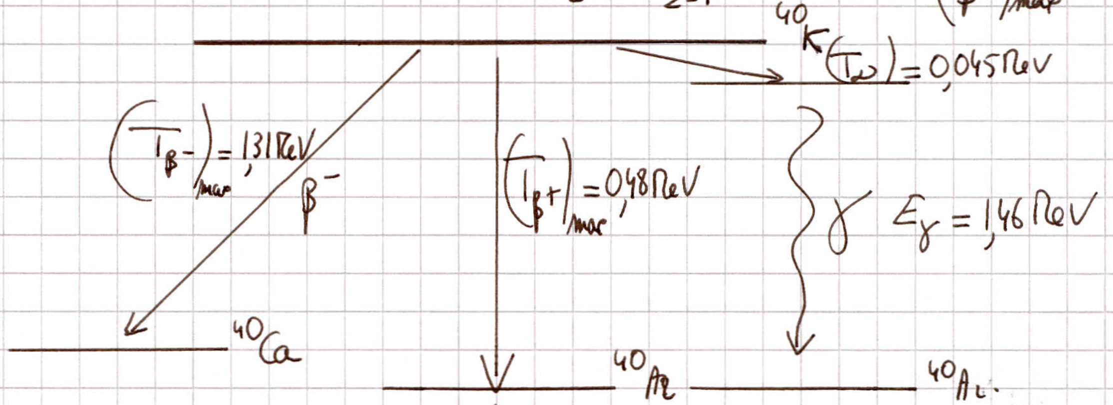
1) β^- $\qquad {}^{40}_{19}K \rightarrow {}^{40}_{20}Ca + {}^{0}_{-1}\beta^- + \bar{\nu}$

β^+ $\qquad {}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + {}^{0}_{+1}\beta^+ + \nu$

CE $\qquad {}^{40}_{19}K + {}^{0}_{-1}e \rightarrow {}^{40}_{18}Ar + \nu$

2) Diagramme des transitions $\qquad$ Cdt $\beta^+ \Rightarrow$ fait en TD

$^{A}_{Z}Mc^2 - {}^{A}_{Z-1}Mc^2 = 2 m_e c^2 + (T_{\beta^+})_{max} + W_{ex}.$



Cdt β^- $\qquad {}^{40}_{19}Mc^2 - {}^{40}_{20}Mc^2 = (T_{\beta^-})_{max}$ $\qquad$ AN: $(T_{\beta^-})_{max} = (39{,}964000 - 39{,}962589) \times 931{,}48 = 1{,}31 \text{ MeV}$

Cdt β^+ $\qquad {}^{40}_{19}Mc^2 - {}^{40}_{18}Mc^2 - 2 m_e c^2 = (T_{\beta^+})_{max}$ $\qquad$ AN: $(T_{\beta^+})_{max} = (39{,}964000 - 39{,}962384) \times 931{,}48 - 1{,}022 = 0{,}48 \text{ MeV}$

Cdt CE $\qquad {}^{40}_{19}Mc^2 - {}^{40}_{18}Mc^2 - E_\gamma = T_\nu$

AN: $(T_\nu) = (39{,}964000 - 39{,}962384) \times 931{,}48 - 1{,}46 = 0{,}05 \text{ MeV}$

3) Période $T_{1/2}$ du ^{40}K $\qquad T_{1/2} = \frac{\ln 2}{\lambda}$ ← constante radioactive.

Calcul du nombre de noyaux N de ^{40}K contenu dans 1g de potassium naturel

1g K(n) $\longrightarrow 1{,}2\ 10^{-4}$ g de $^{40}K = m$

$N = \mathcal{N}_a \frac{m}{A}$ $\qquad$ AN: $N = 1{,}807\ 10^{18}$ noyaux

Désintégration β^- : $\quad A_{\beta^-} = \lambda_{\beta^-} N \quad \Rightarrow \lambda_{\beta^-} = \frac{A_{\beta^-}}{N} \quad$ AN: $\lambda_{\beta^-} = 1{,}53\ 10^{-17}\ s^{-1}$

Capture électronique : $\quad ACE = \lambda_{CE} N \quad \Rightarrow \lambda_{CE} = \frac{ACE}{N} \quad$ AN: $\lambda_{CE} = 1{,}89\ 10^{-18}\ s^{-1}$

$T_{1/2} = \frac{\ln 2}{\lambda_{\beta^-} + \lambda_{CE}} \Rightarrow$ AN: $T_{1/2} = 4\ 10^{16}$ s

$= 1{,}268\ 10^{9}$ ans

4) 0,3g K(n)/kg. $\Rightarrow N = 5{,}42\ 10^{17}$ noyaux/kg.

Désintégration β^- : $\quad A_{\beta^-} = 1{,}53\ 10^{-17} \times 5{,}42\ 10^{17} = 8{,}29\ s^{-1} = 2{,}617\ 10^{8}\ an^{-1}$

$\rightarrow \mathcal{E}_{\beta^-} = 5{,}49\ 10^{-5}$ J

Capture électronique : $A_{CE} = 1{,}89\ 10^{-18} \times 5{,}42\ 10^{17} = 1{,}02\ s^{-1} = 3{,}233\ 10^{7}\ an^{-1}$

$\rightarrow \mathcal{E}_{CE} = 7{,}561\ 10^{-6}$ J

$\Rightarrow \mathcal{E}_{tot} = 6{,}246\ 10^{-5}$ J/kg/an

colossal

cause évasion $\Rightarrow$ élimination du potassium.

☐ Élimination métabolique

^{131}I $\quad$ désintégration β^- $\quad T_{\beta^-} = 8{,}1$ j $\quad ; \lambda_{\beta^-}$

métabolisme $\quad T_m = 180$ j $\quad ; \lambda_m$

1) $dN = -\lambda_\beta N dt - \lambda_m N dt = -(\lambda_{ef}) N dt$ avec $\lambda_{ef} = \lambda_\beta + \lambda_m$

$T_{ef} = \frac{\ln 2}{\lambda_{ef}} = \frac{\ln 2}{\lambda_{\beta^-} + \lambda_m} \Rightarrow \frac{1}{T_{ef}} = \frac{\lambda_{\beta^-}}{\ln 2} + \frac{\lambda_m}{\ln 2} = \frac{1}{T_{\beta^-}} + \frac{1}{T_m}$

2) AN $T_{ef} = \frac{T_\beta T_m}{T_\beta + T_m} \quad \Rightarrow$ AN: 7,75 j $\qquad$ 3 mois $\sim$ 90 j

$N_{\beta^-} = N(0)\, e^{-90\lambda_{\beta^-}}$

$N_{ef} = N(0)\, e^{-90\lambda_{ef}}$

$\frac{N_{ef}}{N_{\beta^-}} = e^{-90\overbrace{(\lambda_{ef} - \lambda_\beta)}^{\lambda_m}} \sim 0{,}707$