Corrigé Pb 1

1) a) $E = \sum_i N_i \varepsilon_i$

b) $\boxed{dE = \sum_i (N_i \, d\varepsilon_i + \varepsilon_i \, dN_i)}$

or $dE = \delta W + \delta Q$, à volume constant $\delta W = -P dV = 0$

et $d\varepsilon_i = 0$ d'où $\boxed{\delta Q = \sum_i \varepsilon_i \, dN_i}$

et enfin $\boxed{\delta W = \sum_i N_i \, d\varepsilon_i}$

2) a) ω = nbre de micro états compatibles avec une configuration $\{N_i\}$

$\boxed{\omega = \frac{N!}{N_1! \dots N_r!}}$ où r est le nombre de niveaux ε_i

b) à l'$\rightleftarrows$, tous les micro état sont équiprobables

or $\sum_{j=1}^{\omega} P(j) = 1 \longrightarrow \boxed{P(j) = \frac{1}{\omega} \quad \forall j}$

$S = -k_B \sum_{j=1}^{\omega} P(j) \ln P(j) = -k_B \sum_{j=1}^{\omega} \frac{1}{\omega} \ln \frac{1}{\omega} = \boxed{+k_B \ln \omega = S}$

c) $\beta = \frac{1}{k_B} \left.\frac{\partial S}{\partial E}\right)_{V,N}$ à volume constant : $\delta W = 0 \rightarrow dE = \delta Q$

$\beta = \frac{1}{k_B} \left.\frac{\partial S}{\partial E}\right)_V \rightarrow dS = k_B \beta \, dE$ (à volume constant)

$= k_B \beta \, \delta Q = \frac{\delta Q}{T}$

d'où $\boxed{\beta = \frac{1}{k_B T}}$

d) $\delta Q = T dS$ avec $\delta Q = \sum_{i=1}^{r} \varepsilon_i dN_i$

* $S = k_B \ln \omega = k_B (\ln N! - \sum_{i=1}^{r} \ln N_i!)$

$dS = -k_B \sum_{i=1}^{r} d(\ln N_i!) \simeq -k_B \sum_{i=1}^{r} d(N_i \ln N_i - N_i)$

$= -k_B \left(\sum_i (dN_i) \ln N_i\right)$ $(\sum_i N_i = N = C^{te})$

$dS = -k_B \sum_i \ln N_i \, dN_i$ ①

* $T dS = \delta Q = \sum_{i=1}^{r} \varepsilon_i \, dN_i \rightarrow dS = \frac{1}{T} \sum_i \varepsilon_i \, dN_i$ ②

① - ② $\rightarrow \sum_i (\varepsilon_i + kT \ln N_i) \, dN_i = 0$

mais ici les r variables N_i sont dépendantes

car $\sum_i N_i = N \Rightarrow \sum_i dN_i = 0$

r variables } $r-1$ variables indépendantes
1 contrainte }

$$\sum_{i=1}^{r}(\varepsilon_i + kT\ln N_i)\,dN_i = 0 = \sum_{i=1}^{r-1}(\varepsilon_i + kT\ln N_i)\,dN_i + (\varepsilon_r + kT\ln N_r)\,dN_r$$

avec $dN_r + \sum_{i=1}^{r-1} dN_i = 0 \rightarrow dN_r = -\sum_{i=1}^{r-1} dN_i$

$$\rightarrow 0 = \sum_{i=1}^{r-1}\left[(\varepsilon_i - \varepsilon_r) + kT(\ln N_i - \ln N_r)\right] dN_i$$ ici les $r-1$ variables sont indépendantes

$$\Rightarrow (\varepsilon_i - \varepsilon_r) + kT(\ln N_i - \ln N_r) = 0 \quad \forall i = 1, \dots r-1$$

$$\Rightarrow \ln N_i + \frac{\varepsilon_i}{k_BT} = \ln N_r + \frac{\varepsilon_r}{k_BT} = \text{cte} = C$$

d'où $N_i = \alpha\, e^{-\varepsilon_i/k_BT}$; $\sum_i N_i = N = \alpha \sum_i e^{-\varepsilon_i/kT}$

$\rightarrow \alpha = \dfrac{N}{\sum_i e^{-\varepsilon_i/k_BT}}$.

$$\boxed{N_i = \frac{N}{Z} e^{-\beta\varepsilon_i}}$$ avec $Z = \sum_i e^{-\beta\varepsilon_i}$; $\beta = \dfrac{1}{k_BT}$

3) $\beta\,\delta Q = N\,d(\ln z) + d(\beta E)$

a) transformation adiabatique $\delta Q = 0 \rightarrow d(\beta E) = -N\,d(\ln z)$

or, il s'agit de gaz parfait : $E\beta = \frac{3}{2}N$ $\quad (E = \frac{3}{2}NkT)$

d'où $d(\beta E) = 0 \longrightarrow d(\ln z) = 0 \longrightarrow \underline{z = \text{cte}}$

b) $z = \dfrac{V}{\lambda_{th}^3} = \dfrac{V}{h^3}\left(\dfrac{m\,kT}{2\pi}\right)^{3/2} = \text{cte} \Rightarrow \underline{V\,T^{3/2} = \text{cte}}$

or $PV = NkT \Rightarrow PV\,V^{2/3} = NkT\,V^{2/3} = \text{cte}$

c) $\rightarrow PV^{5/3} = \text{cte} \rightarrow \underline{\gamma = 5/3}$.

bozon

d) $dE = \delta Q + \delta W \rightarrow \beta\,\delta Q = \beta\,dE - \beta\,\delta W = \underline{d(\beta E) - E\,d\beta - \beta\,\delta W}$

or $\delta W = \sum_i N_i\,d\varepsilon_i = \dfrac{N}{Z}\sum_i e^{-\beta\varepsilon_i}d\varepsilon_i \rightarrow \underline{\beta\,\delta W = \dfrac{N}{Z}\sum_i e^{-\beta\varepsilon_i}(\beta\,d\varepsilon_i)}$

$d(e^{-\beta\varepsilon_i}) = -d(\beta\varepsilon_i)\,e^{-\beta\varepsilon_i} = -d\beta\,\varepsilon_i\,e^{-\beta\varepsilon_i} - \beta(d\varepsilon_i)\,e^{-\beta\varepsilon_i}$

$\beta\,\delta W = \dfrac{N}{Z}\sum_i(\beta\,d\varepsilon_i)\,e^{-\beta\varepsilon_i} = \dfrac{N}{Z}\sum_i\left\{(-d\beta)\,\varepsilon_i\,e^{-\beta\varepsilon_i} - d(e^{-\beta\varepsilon_i})\right\}$

$\rightarrow -\beta\,\delta W = \underbrace{\dfrac{N}{Z}\sum_i(\varepsilon_i\,e^{-\beta\varepsilon_i})}_{E}\,d\beta + \dfrac{N}{Z}\,d\underbrace{\left(\sum_i e^{-\beta\varepsilon_i}\right)}_{Z}$

$= E\,d\beta + \dfrac{N}{Z}dZ = E\,d\beta + N\,d\ln Z$

Enfin $\boxed{\beta\,\delta Q = d(\beta E) + N\,d(\ln Z)}$

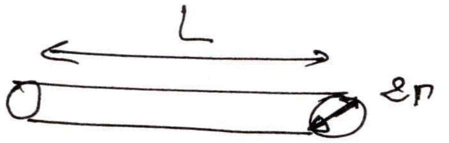
III

1) $\vec{J}_n = -D\,\underbrace{\overrightarrow{grad}\, n_v}_{m^{-4}} = -D \dfrac{\partial n_v}{\partial x}\vec{e_x}$ → densité particulaire (m

coefficient de diffusion $\left(\dfrac{s^{-1}m^{-2}}{m^{-4}} \equiv s^{-1}m^{+2}\right)$

vecteur courant volumique de particules $(s^{-1}m^{-2})$

2)



$S_p = \pi r^2$

$$\Phi_p = \iint_S \vec{J}_n \cdot \vec{dS} = \iint_S \vec{J}_n \cdot \vec{n}\, dS$$

$$\Phi_p = \iint_S -D \frac{\partial n_v}{\partial x} (\vec{e_x} \cdot \vec{n})\, dS$$

$\vec{n} = -\vec{e_x}$ $\qquad \Phi_p = +D \dfrac{dn_v}{dx} \iint_S dS$

$$\Phi_p = D \frac{dn_v}{dx} \pi r^2$$

$$\frac{dn_v}{dx} \simeq \frac{n_{2v}(t) - n_{1v}(t)}{e} = -\frac{\Delta(t)}{e}$$

$$\Phi_p \simeq -D \frac{\Delta(t)}{e} \pi r^2 \Rightarrow .$$

3) $\Phi_m = p \times S \times \Phi_p = -\dfrac{D \pi r^2 p S}{e} \Delta(t)$

$= -K \Delta(t) S$ avec $K = \dfrac{p \pi r^2 D}{e}$

$$\Rightarrow r = \sqrt{\frac{eK}{p\pi D}} = \sqrt{\frac{10^{-5} \times 10^{-6}}{10^{10} \times \pi\, 10^{-9}}} = 0{,}56\,\mu m$$

4) $$-\frac{dN_1(t)}{dt} = \frac{dN_2(t)}{dt} = \phi_m = -KS\Delta(t)$$

5) $$\frac{1}{V_1}\frac{dN_1(t)}{dt} = \frac{dn_{1v}(t)}{dt} \qquad \frac{1}{V_2}\frac{dN_2(t)}{dt} = \frac{dn_{2v}(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{d(n_{1v}(t) - n_{2v}(t))}{dt} = \frac{dn_{1v}(t)}{dt} - \frac{dn_{2v}(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{1}{V_1}\frac{dN_1(t)}{dt} - \frac{1}{V_2}\frac{dN_2(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{KS(\Delta(t)}{V_1} + \frac{KS\,\Delta t)}{V_2} = KS\,\Delta(t)\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

$$\boxed{\frac{d(\Delta t)}{dt} - \frac{\Delta(t)}{\tau} = 0}$$

$$\frac{d(\Delta t)}{\Delta(t)} = -\frac{dt}{\tau} \Rightarrow \Delta(t) = C\,e^{-\alpha t}$$

$$\alpha = \frac{1}{\tau}$$

$$\Delta(t=0) = \Delta_0 = C$$

$$\Delta(t) = \Delta_0\, e^{-t/\tau}$$

t_1 tq $$\Delta(t=t_1) = \frac{\Delta_0}{10} = \Delta_0\, e^{-t_1/\tau}$$

$$\Rightarrow e^{t_1/\tau} = 10 \Rightarrow t_1/\tau = \ln 10$$

$$t_1 = \tau \ln 10 = 2{,}3\,\tau$$