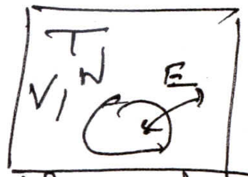
Corrigé juin 2008 - Physique Stat

I)

1) E fluctue

1 $T = cte$, $V = cte$, $N = cte$



2) pour tout système en équilibre thermique avec un thermostat à la température T la contribution à l'énergie moyenne de chaque variable intervenant au carré et seulement au carré, dans l'expression de l'énergie, vaut

1 $1/2\, k_B T$

$$E = \overset{k_BT/2}{a p^2} + \overset{k_BT/2}{b q^2}$$

3) $\langle E \rangle \overset{?}{=}$ $E = E_{trans} + E_{rot} = H_{trans} + H_{rot}$

1 $$E = \frac{p_x^2}{2M} + \frac{p_y^2}{2m} + \frac{p_z^2}{2M} + \frac{p_\theta^2}{2I} + \frac{p_\varphi^2}{2I\sin^2\theta}$$

$\langle E \rangle = \frac{5}{2} k_B T$ pour 1 particule

pour les N molécules $\langle E \rangle = N\langle \varepsilon \rangle = \frac{5}{2} N k_B T$

1 $$C_V = \left(\frac{\partial \langle E \rangle}{\partial T}\right)_{V,N} = \frac{5}{2} N k_B$$

$$C_p - C_V = N k_B \Rightarrow C_p = \frac{7}{2} N k_B$$

1 $$\gamma = \frac{C_p}{C_V} = \frac{7}{5} = 1{,}4$$

II

1) $\begin{cases} z = e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2} = 1 + e^{-\beta\varepsilon} \\ Z = z^N = (1 + e^{-\beta\varepsilon})^N \end{cases}$

2) $P_1 = \frac{1}{z} e^{-\beta\varepsilon_1} = \frac{1}{1+e^{-\beta\varepsilon}}$, $P_2 = \frac{1}{z} e^{-\beta\varepsilon_2} = \frac{e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}}$

$P_1 + P_2 = 1$

$P_2 = \frac{1}{e^{\beta\varepsilon}+1}$

3) $N_1 = P_1 N = \frac{N}{1+e^{-\beta\varepsilon}}$

$N_2 = P_2 N = \frac{N}{e^{\beta\varepsilon}+1}$ $\qquad N_1 + N_2 = N$

4) θ tq $\frac{\varepsilon}{k_B T} = \beta\varepsilon = \frac{\theta}{T}$

5) $T << \theta$ Basses Températures $\qquad N_1 = \frac{N}{1+e^{-\frac{\theta}{T}}}$ $\quad N_2 = \frac{N}{e^{\frac{\theta}{T}}+1}$

$N_1 \to N$

$N_2 \to N e^{-\theta/T} \to 0$

$\to$ Toutes les molécules sont au niveau le + bas

$T >> \theta$ Hautes températures $\qquad \frac{\varepsilon}{k_B T} << 1 \qquad \varepsilon << k_B T$

$N_1 \to \frac{N}{1 + 1 - \frac{\theta}{T}} \sim \frac{N}{2}$

$N_2 \to \frac{N}{2 + \frac{\theta}{T}} \sim \frac{N}{2}$ les molécules se repartissent de manière égale sur les 2 niveaux

écart entre les 2 niveaux est négligeable

6) $\langle E \rangle = N_1 \varepsilon_1 + N_2 \varepsilon_2 = \frac{N\varepsilon}{e^{\beta\varepsilon}+1}$

$\langle E \rangle = -\frac{\partial}{\partial\beta}(\ln Z) = -N\frac{\partial}{\partial\beta}(\ln z) = N\frac{\varepsilon e^{-\beta\varepsilon}}{1+e^{-\beta\varepsilon}} = \frac{N\varepsilon}{e^{\beta\varepsilon}+1}$

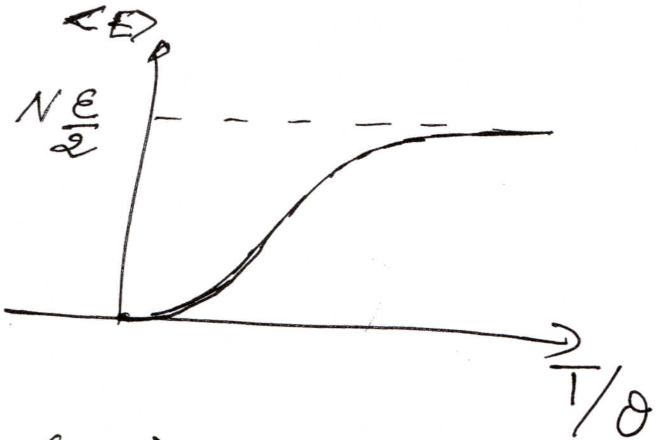
$$\langle E\rangle = \frac{N\varepsilon}{e^{\frac{\theta}{T}}+1} = Nk_B\frac{\theta}{e^{\theta/T}+1}$$

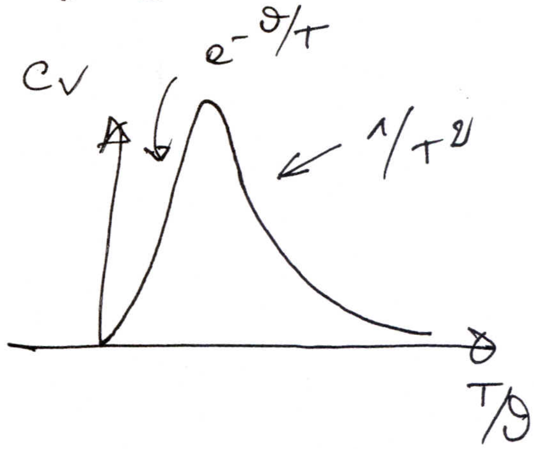
$\frac{T}{\theta} \gg 1$ $\quad T \gg \theta \quad$ Hte température

$$\langle E\rangle \to \frac{N\varepsilon}{1+\frac{\theta}{T}} \sim \frac{N\varepsilon}{2}$$

$\frac{T}{\theta} \ll 1$ $\quad T \ll \theta \quad$ Basse température

$$\langle E\rangle \to N\varepsilon\, e^{-\theta/T} \to 0$$





7) $C_V = \left(\frac{\partial\langle E\rangle}{\partial T}\right)_{V,N} = \left(\frac{\partial\langle\varepsilon\rangle}{\partial\beta}\right)_{V,N}\left(\frac{\partial\beta}{\partial T}\right)_{V,N} = -\frac{1}{k_BT^2}\left(\frac{\partial\langle E\rangle}{\partial\beta}\right)_{V,N}$

$$C_V = -\frac{k_B N}{(k_BT)^2}\cdot\left(\frac{-\varepsilon^2 e^{\beta\varepsilon}}{(e^{\beta\varepsilon}+1)^2}\right) = \frac{Nk_B}{(k_BT)^2}\frac{\varepsilon^2 e^{\beta\varepsilon}}{(1+e^{\beta\varepsilon})^2}$$

$$C_V = Nk_B\left(\frac{\varepsilon}{k_B}\right)^2\frac{1}{T^2}\frac{e^{\beta\varepsilon}}{(1+e^{\beta\varepsilon})^2}$$

$$C_V = Nk_B\left(\frac{\theta}{T}\right)^2\frac{e^{\beta\varepsilon}}{(1+e^{\beta\varepsilon})^2} = Nk_B\left(\frac{\theta}{T}\right)^2\frac{e^{+\theta/T}}{(1+e^{\theta/T})^2}$$

$T \to 0 \quad (T \ll \theta)$

$$C_V \to Nk_B\left(\frac{\theta}{T}\right)^2 e^{-\theta/T} \to 0$$

$T \to \infty \quad (T \gg \theta)$

$$C_V \to Nk_B\left(\frac{\theta}{T}\right)^2\frac{(1+\theta/T)}{(2+\theta/T)^2}$$

$$C_V \to \frac{Nk_B}{1}\left(\frac{\theta}{T}\right)^2 \sim \alpha\frac{1}{T^2} \to 0$$

III

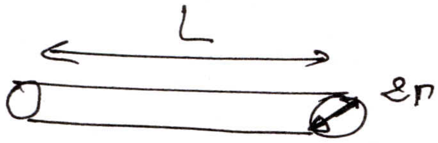
1) $\vec{J}_n = -D \underbrace{\overrightarrow{grad}\, n_v}_{m^{-4}} = -D \frac{\partial n_v}{\partial x} \vec{e_x}$

n_v : densité particulaire (m^{-3})

D : coefficient de diffusion $\left(\frac{s^{-1} m^{-2}}{m^{-4}} \equiv s^{-1} m^{+2} \right)$

$\vec{J}_n$: vecteur courant volumique de particules ($s^{-1} m^{-2}$)

2)



$S_p = \pi r^2$

$\Phi_p = \iint_S \vec{J}_n \cdot \vec{dS} = \iint_S \vec{J}_n \cdot \vec{n}\, dS$

$\Phi_p = \iint_S -D \frac{\partial n_v}{\partial x} (\vec{e_x} \cdot \vec{n})\, dS$

$\vec{n} \equiv -\vec{e_x}$ $\qquad \Phi_p = +D \frac{d n_v}{dx} \iint_S dS$

$\Phi_p = D \frac{d n_v}{dx} \pi r^2$

$\frac{d n_v}{dx} \simeq \frac{n_{2v}(t) - n_{1v}(t)}{e} = -\frac{\Delta(t)}{e}$

$\Phi_p \simeq -D \frac{\Delta(t)}{e} \pi r^2 \Rightarrow$.

3) $\Phi_m = p \times S \times \Phi_p = -\frac{D \pi r^2 p S}{e} \Delta(t)$

$= -K \Delta(t) S$ $\qquad$ avec $K = \frac{p \pi r^2 D}{e}$

$\Rightarrow r = \sqrt{\frac{eK}{p \pi D}} = \sqrt{\frac{10^{-5} \times 10^{-6}}{10^{10} \times \pi\, 10^{-9}}} = 0,56\ \mu m$

4) $$-\frac{dN_1(t)}{dt} = \frac{dN_2(t)}{dt} = \Phi_m = -KS\Delta(t)$$

5) $$\frac{1}{V_1}\frac{dN_1(t)}{dt} = \frac{dn_{1v}(t)}{dt} \qquad \frac{1}{V_2}\frac{dN_2(t)}{dt} = \frac{dn_{2v}(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{d(n_{1v}(t) - n_{2v}(t))}{dt} = \frac{dn_{1v}(t)}{dt} - \frac{dn_{2v}(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{1}{V_1}\frac{dN_1(t)}{dt} - \frac{1}{V_2}\frac{dN_2(t)}{dt}$$

$$\frac{d(\Delta t)}{dt} = \frac{KS\Delta(t)}{V_1} + \frac{KS\Delta(t)}{V_2} = KS\Delta(t)\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$$

$$\boxed{\frac{d(\Delta t)}{dt} - \frac{\Delta(t)}{\tau} = 0}$$

$$\frac{d(\Delta t)}{\Delta(t)} = -\frac{dt}{\tau} \Rightarrow \Delta(t) = C\,e^{-\alpha t}$$

$$\alpha = \frac{1}{\tau}$$

$$\Delta(t=0) = \Delta_0 = C$$

$$\Delta(t) = \Delta_0\,e^{-t/\tau}$$

t_1 tq $$\Delta(t=t_1) = \frac{\Delta_0}{10} = \Delta_0\,e^{-t_1/\tau}$$

$$\Rightarrow e^{t_1/\tau} = 10 \Rightarrow t_1/\tau = \ln 10$$

$$t_1 = \tau \ln 10 = 2{,}3\,\tau$$