Corrigé examen Thermo Statistique
Décembre 2008

I)

Rendement d'une lampe

1) corps noir : cavité fermée contenant un gaz de photons en équilibre thermique avec un thermostat

un corps noir absorbe intégralement le rayonnement qu'il reçoit et ne réfléchit rien.

2) $$u(\lambda,T) = \frac{8\pi hc}{\lambda^5}\,\frac{1}{e^{hc/\lambda k_B T}-1}$$

$$x = \frac{hc}{\lambda k_B T} \qquad \lambda = \frac{hc}{k_B T x} \qquad d\lambda = \frac{hc}{k_B T}\,\frac{dx}{x^2}$$

$$u_{totale}(T) = \frac{8\pi (hc)}{(hc)^5}\int_0^\infty \frac{x^3\,dx}{e^x-1} \times (k_B T)^4$$

$$u_{totale}(T) = \frac{8\pi}{h^3c^3}(k_B T)^4 \int_0^\infty \frac{x^3}{e^x-1}\,dx$$

$$E(T) = V\,u_{totale} = \frac{8\pi V (k_B T)^4}{h^3c^3}\,\frac{\pi^4}{15}$$

$$r = \frac{E_V(T)}{E(T)} = \frac{V\int_{\lambda_1}^{\lambda_2} u(\lambda,T)\,d\lambda}{V\int_0^\infty u(\lambda,T)\,d\lambda}$$

$$r = \frac{\int_{x_1}^{x_2}\frac{x^3}{e^x-1}\,dx}{\int_0^\infty \frac{x^3}{e^x-1}\,dx} \qquad x_1 = \frac{hc}{\lambda_1 k_B T} \qquad x_2 = \frac{hc}{\lambda_2 k_B T}$$

$$r = \frac{15}{\pi^4}\int_{x_1}^{x_2}\frac{x^3}{e^x-1}\,dx$$

$T = 2500\,K$

$x_1 = 7{,}2$

$x_2 = 14{,}5$

$$r = \frac{15}{\pi^4}\times\left[-e^{-x_2}(x_2^3+3x_2^2+6x_2-6) + e^{-x_1}(x_1^3+3x_1^2+6x_1+6)\right]$$

$$(-0{,}019+0{,}431)\times 15/\pi^4 = 0{,}066$$

3) $r \nearrow$ si $T \nearrow$

II Vibration d'un solide - modèle d'Einstein

1) $$z = \sum_{n_x,n_y,n_z=0}^{\infty} \exp -\beta \varepsilon_{n_x,n_y,n_z} = \sum_{n_x,n_y,n_z=0}^{\infty} \exp -\beta\left[\left(n_x+\frac{1}{2}\right)+\left(n_y+\frac{1}{2}\right)+\left(n_z+\frac{1}{2}\right)\right]\hbar$$

$$z = \sum_{n_x=0}^{\infty} \exp -\beta \varepsilon_{n_x} \sum_{n_y=0}^{\infty} \exp -\beta \varepsilon_{n_y} \sum_{n_z=0}^{\infty} \exp -\beta \varepsilon_{n_z}$$

$$z = \left(\sum_{n_x=0}^{\infty} \exp -\beta\left(n_x+\frac{1}{2}\right)\hbar\omega\right)^3$$

$$z = \left[\exp\left(-\beta\frac{\hbar\omega}{2}\right) \sum_{n_x=0}^{\infty} \exp(-\beta n_x \hbar\omega)\right]^3$$

$$\sum_{n_x=0}^{\infty} \exp(-\beta n_x \hbar\omega) = 1 + \exp(-\beta\hbar\omega) + \exp(-2\beta\hbar\omega) + \dots$$

somme d'une série ~~+ exp(~~ géométrique de raison $\exp(-\beta\hbar\omega)$

$$z = \left(e^{-\beta\frac{\hbar\omega}{2}}\right)^3 \left(\frac{1}{1-e^{-\beta\hbar\omega}}\right)^3$$

$$Z = z^N = \left(e^{-\beta\frac{\hbar\omega}{2}}\right)^{3N} \left(\frac{1}{1-e^{-\beta\hbar\omega}}\right)^{3N}$$

$$Z = e^{-\beta\frac{3}{2}N\hbar\omega} \frac{1}{(1-e^{-\beta\hbar\omega})^{3N}}$$

$$\langle \varepsilon \rangle = -\frac{\partial}{\partial\beta} \ln z = \frac{3}{2}\hbar\omega + \frac{3\hbar\omega\, e^{-\beta\hbar\omega}}{1-e^{-\beta\hbar\omega}}$$

$$\langle \varepsilon \rangle = \frac{3}{2}\hbar\omega + \frac{3\hbar\omega}{e^{\beta\hbar\omega}-1}$$

$$\langle E \rangle = N\langle \varepsilon \rangle$$

$$F = -k_B T \ln Z = N\left[\frac{3}{2}\hbar\omega + 3k_B T \ln(1-e^{-\beta\hbar\omega})\right]$$

$$C_V = \left(\frac{\partial\langle E\rangle}{\partial T}\right)_V = \left(\frac{\partial\langle E\rangle}{\partial\beta}\right)_V \frac{\partial\beta}{\partial T} = -\frac{1}{k_B T^2}\left(\frac{\partial\langle E\rangle}{\partial\beta}\right)_V$$

$$C_V = \frac{3N\hbar\omega}{k_B T^2} \frac{\hbar\omega\, e^{+\beta\hbar\omega}}{(e^{+\beta\hbar\omega}-1)^2} = 3Nk_B\left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega}-1)^2}$$

$$\theta_E = \frac{\hbar \omega}{k_B}$$

$$C_V = 3Nk_B \left(\frac{\theta_E}{T}\right)^2 \frac{e^{+\theta_E/T}}{\left(e^{\theta_E/T} - 1\right)^2}$$

$T > \theta_E$ Hauts températures $\qquad \frac{\theta_E}{T} < 1$

$$e^{\theta_E/T} \sim 1 + \frac{\theta_E}{T}$$

$$e^{\theta_E/T} - 1 \sim \frac{\theta_E}{T}$$

$$C_V \simeq 3Nk_B \left(\frac{\theta_E}{T}\right)^2 \frac{\left(1 + \theta_E/T\right)}{\left(\theta_E/T\right)^2}$$

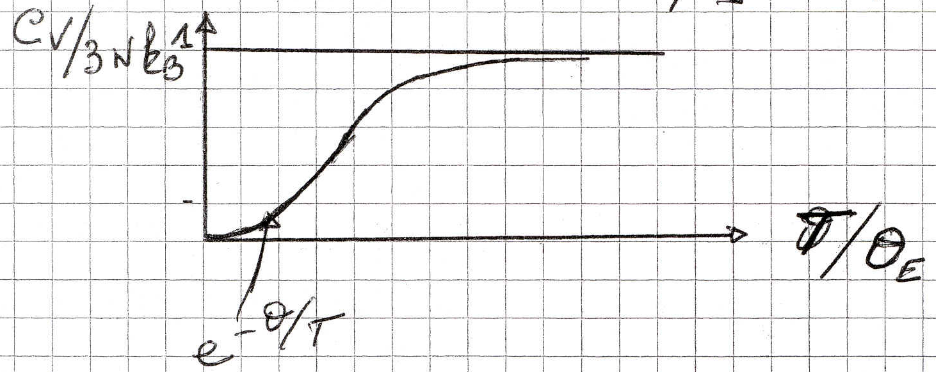
$$C_V \simeq 3Nk_B \simeq 3Nk_B$$

$T < \theta_E$ basses températures $\qquad \frac{\theta_E}{T} > 1$

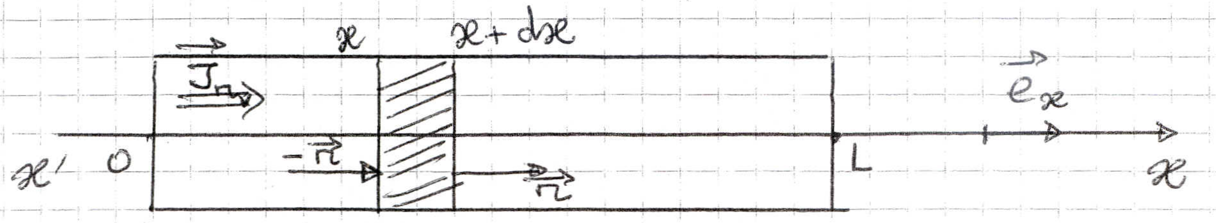
$$e^{\theta_E/T} - 1 \sim e^{\theta_E/T}$$

$$C_V \simeq 3Nk_B \left(\frac{\theta_E}{T}\right)^2 e^{-\theta_E/T}$$

tend vers 0 avec T de façon exponentielle



III Diffusion axiale de molécules



1) Loi de Fick

$$\vec{J_{n_v}} = -D \frac{dn_v(x)}{dx} \vec{e_x}$$

vecteur courant volumique de molécules en $m^{-2}s^{-1}$

coéfficient de diffusion en $m^2 s^{-1}$

gradient de densité en m^{-4}

2) $n_v(x=0) = n_1$ $\qquad$ $n_v(x=L) = n_2$

$$dN^r = dt \iint_S \vec{J_{n_v}} \cdot (-\vec{n})\, dS \qquad dN^P = 0$$

$$(dn_v)\, S\, dx = \left[S\, J_{n_v}(x) - S\, J_{n_v}(x+dx)\right] dt$$

$$= -\left(\frac{\partial J_{n_v}}{\partial x}\right) dx\, dt$$

$$\frac{dn_v}{dt} = -\frac{\partial J_{n_v}}{\partial x} = -\operatorname{div} \vec{J_n}$$

3) $$\frac{dn_v}{dt} = -\frac{\partial}{\partial x}\left(-D \frac{dn_v}{dx}\right) = -D \frac{d^2 n_v}{dx^2} = 0$$

$$\frac{d^2 n_v}{dx^2} = 0$$

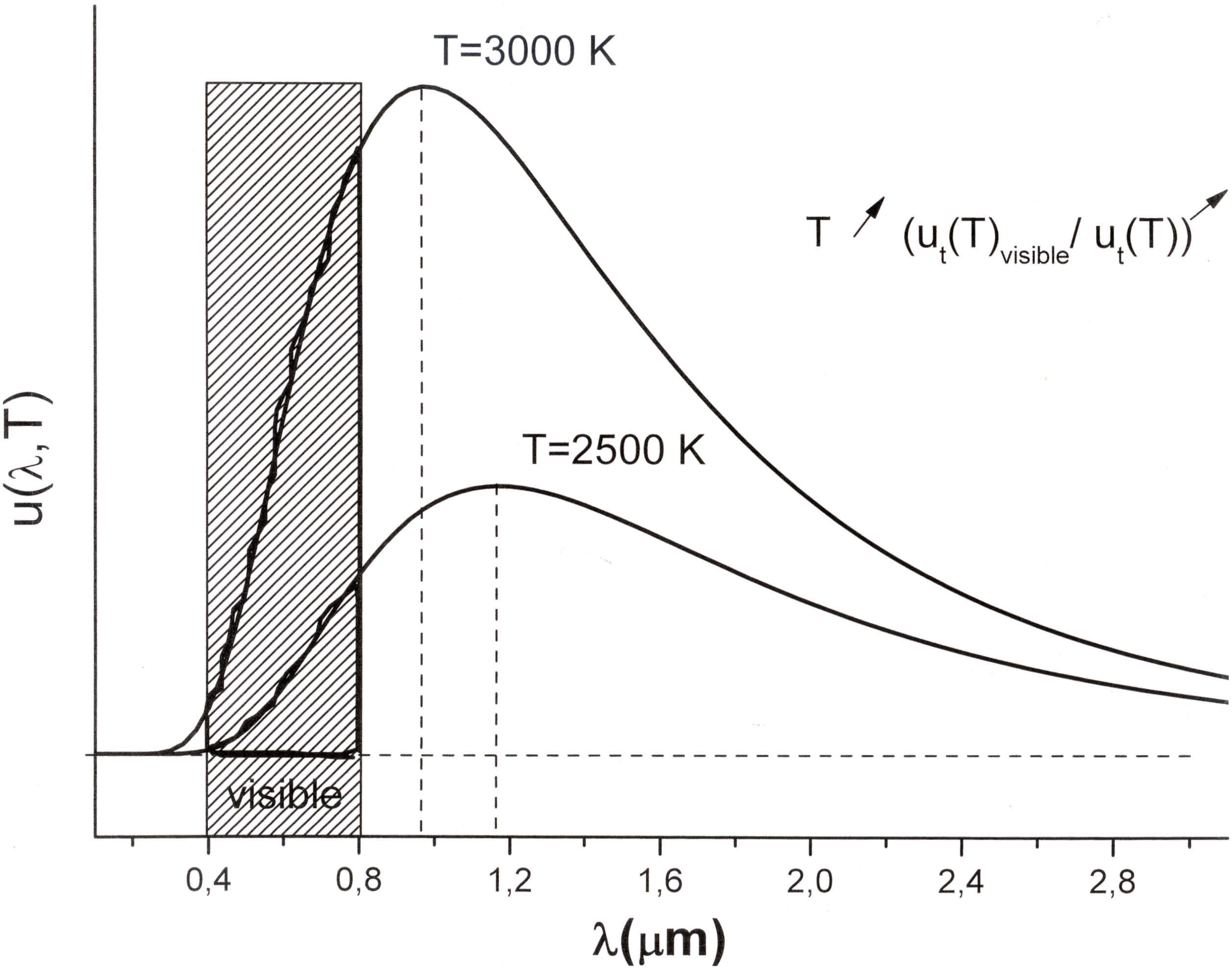
4) $$\frac{dn_v}{dx} = cte = -\frac{J_{n_v}}{D} \qquad n_v(x) = -\frac{J_{n_v}}{D} x + cte$$

$$n_v(0) = cte = n_1 \qquad n_v(L) = -\frac{J_{n_v}}{D} L + cte$$

$$n_v(L) = -\frac{J_{n_v} L}{D} + n_v(0)$$

$$n_2 - n_1 = -\frac{J_{n_v} L}{D} \qquad \frac{n_2 - n_1}{L} = -\frac{J_{n_v}}{D}$$

$$n_v(x) = \frac{(n_2 - n_1)}{L} x + n_1$$



u(λ,T)
T=3000 K
T=2500 K
T ↗ (u_t(T)_visible / u_t(T)) ↗
visible
0,4
0,8
1,2
1,6
2,0
2,4
2,8
λ(μm)