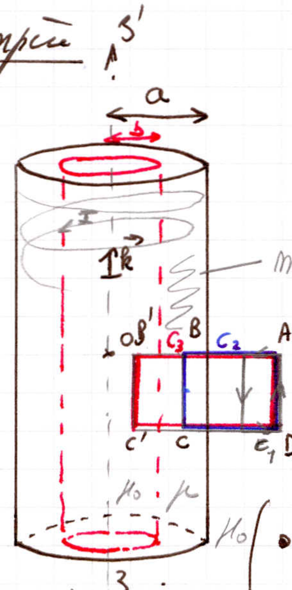
Théorème d'Ampère



n spires par mètres

1) Th - rappel. $\displaystyle\oint_C \vec{H}\cdot\vec{dl} = I_{ex}$

vecteur excitation magnétique

intensité extérieure.

1) $\vec{B}$ et $\vec{H}$ axe z'oz

$\vec{H}$

- $r > a$ $\qquad \vec{B} = \vec{0}$. $\vec{H} = \vec{0}$
- $b < r < a$

$$\int_{C_2} \vec{H}\cdot\vec{dl} = \cancel{\int_A^B \vec{H}\cdot\vec{dl}} + \underbrace{\int_B^C \vec{H}\cdot\vec{dl}}_{H(r)\times BC} + \cancel{\int_C^D \vec{H}\cdot\vec{dl}} + \cancel{\int_D^A \vec{H}\cdot\vec{dl}}$$

$\vec{H}\perp\vec{dl}$ $\qquad \vec{H}\perp\vec{dl}$ $\qquad \vec{H}=\vec{0}$

$$= H(r)\times BC = n \times BC \times I$$

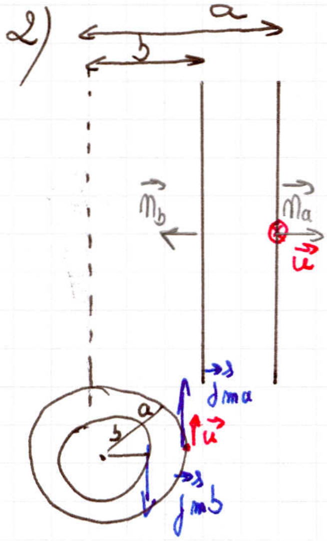
$$\Rightarrow \vec{H}(r) = n I \vec{k}$$

- $r < b$ m̂ raisonnement $\Rightarrow \vec{H}(r) = n I \vec{k}$

on en déduit : $\vec{B}$

- $r > a$ $\qquad \vec{B} = \vec{0}$
- $b < r < a$ $\qquad \vec{B} = \mu\vec{H} = \mu n I \vec{k}$
- $r < b$ $\qquad \vec{B} = \mu_0\vec{H} = \mu_0 n I \vec{k}$

2)



Volume $\vec{j}_m = \vec{rot}\,\vec{\Pi} = \vec{0}$ ($\vec{H}$ et $\vec{B}$ colinéaires, axiaux) $\Rightarrow \vec{\Pi}$ aussi

$\Rightarrow$ pas d'aimantation en volume

surface $\vec{j}^{\,s}_m = \vec{\Pi}\wedge\vec{n}$.

$$\vec{B} = \mu_0(\vec{H}+\vec{\Pi}) = \mu\vec{H} \text{ avec } \vec{H} = nI\vec{k}$$

soit $\displaystyle\vec{\Pi} = \frac{\mu-\mu_0}{\mu_0}\vec{H}$

$$\vec{j}^{\,s}_{ma} = \vec{\Pi}\wedge\vec{n}_a = \frac{\mu-\mu_0}{\mu_0} nI \underbrace{\vec{k}\wedge\vec{n}_a}_{\vec{u}} = \frac{\mu-\mu_0}{\mu_0} nI\vec{u}$$

$$\vec{j}^{\,s}_{mb} = \vec{\Pi}\wedge\vec{n}_b = \frac{\mu-\mu_0}{\mu_0} nI \underbrace{\vec{k}\wedge\vec{n}_b}_{-\vec{u}} = -\vec{j}^{\,s}_{ma}$$

Corrigé Electromagnétisme L3 Sept 2001

<u>Atomes de Thomson</u> : oscillateurs m, q, ω_0, densité volumique N

$$\vec{E} = \vec{E}_0 e^{i(\vec{k}.\vec{r} - \omega t)} \quad \text{chp macroscopique moyen.}$$

1) $$\vec{\rho} = \vec{\rho}_0 e^{i(\vec{k}.\vec{r} - \omega t)} \quad \text{déplacement.}$$

Equadiff du mouv^t

$$m\ddot{\rho} = -k\rho + qE \qquad E \text{ chp moyen}$$

$$\ddot{\rho} + \underbrace{\frac{k}{m}}_{\omega_0^2}\rho = q\frac{E}{m}$$

- Milieu LHI $\quad \vec{P} = \varepsilon_0 \chi \vec{E} \qquad \vec{P} = Nq\vec{\rho}$ (fct du déplacement)
- Susceptibilité χ

$$\rho = \rho_0 e^{i(\vec{k}.\vec{r} - \omega t)} \Rightarrow \ddot{\rho} = -\omega^2 \rho$$

$$-\omega^2\rho_0 + \omega_0^2\rho_0 = q\frac{E_0}{m} \Rightarrow E_0 = \frac{m\rho_0(\omega_0^2 - \omega^2)}{q}$$

$$\varepsilon_0 \chi E_0 = \varepsilon_0 \chi \left(\frac{m\cancel{\rho_0}(\omega_0^2 - \omega^2)}{q}\right) = Nq\cancel{\rho_0}$$

$$\Rightarrow \chi = \frac{Nq^2}{\varepsilon_0 m(\omega_0^2 - \omega^2)}$$

- <u>Indice de réfraction</u> $\quad n^2 = \varepsilon_r = 1 + \chi \quad \Rightarrow n^2 = 1 + \dfrac{Nq^2}{\varepsilon_0 m(\omega_0^2 - \omega^2)}$

2) Présence des proches voisins $\quad \vec{E} \longrightarrow \vec{E_\ell} = \vec{E} + \dfrac{\vec{P}}{3\varepsilon_0}$

Eq diff $$\ddot{\rho} + \omega_0^2\rho = \frac{q}{m}\left(E + \frac{P}{3\varepsilon_0}\right) = \frac{q}{m}\left(E + \frac{Nq\rho}{3\varepsilon_0}\right)$$

$$\ddot{\rho} + \underbrace{\left(\omega_0^2 - \frac{Nq^2}{3m\varepsilon_0}\right)}_{\Omega_0^2}\rho = q\frac{E}{m} \qquad \text{eq diff équivalente à la précédente}$$

$$\Omega_0^2 = \omega_0^2 - \frac{Nq^2}{3m\varepsilon_0}$$

②

3) $\mathrm{div}\,\vec{S} + \dfrac{dW}{dt} = 0$ ou $\vec{\nabla}.\vec{S} + \dfrac{dw}{dt} = 0$ — $\vec{S} = \vec{E} \wedge \vec{H}$ vecteur de Poynting, w densité d'énergie

Maxwell-Faraday $\vec{\nabla}\wedge\vec{E} = \vec{\mathrm{rot}}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

Maxwell-Ampère $\vec{\nabla}\wedge\vec{H} = \vec{\mathrm{rot}}\,\vec{H} = \dfrac{\partial \vec{D}}{\partial t}$ en absence de courant volumique extérieur et d'aimantation.

$$\vec{\nabla}.\vec{S} = \vec{\nabla}.(\vec{E}\wedge\vec{H}) = \vec{H}\cdot\vec{\nabla}\wedge\vec{E} - \vec{E}\cdot\vec{\nabla}\wedge\vec{H}$$ rappel texte

$$= -\vec{H}.\frac{\partial \vec{B}}{\partial t} - \vec{E}.\frac{\partial \vec{D}}{\partial t}$$ Milieu amagnétique $\vec{B} = \mu_0 \vec{H}$ (pas d'aimantation volumique)

$$= -\mu_0 \vec{H}.\frac{\partial \vec{H}}{\partial t} - \vec{E}.\frac{\partial}{\partial t}(\varepsilon_0 \vec{E} + \vec{P}) = \underbrace{-\mu_0 \vec{H}.\frac{\partial \vec{H}}{\partial t} - \varepsilon_0 \vec{E}.\frac{\partial \vec{E}}{\partial t}} - \vec{E}.\frac{\partial \vec{P}}{\partial t}$$

$$= -\frac{1}{2}\left[\varepsilon_0 \frac{\partial \vec{E}^2}{\partial t} + \mu_0 \frac{\partial \vec{H}^2}{\partial t}\right] - \vec{E}.\frac{\partial \vec{P}}{\partial t}$$

$$= -\frac{dW}{dt}$$

donc $$\frac{dw}{dt} = \frac{1}{2}\left[\varepsilon_0 \frac{\partial E^2}{\partial t} + \mu_0 \frac{\partial H^2}{\partial t}\right] + \vec{E}.\frac{\partial \vec{P}}{\partial t}$$ cqfd.

$$= \underbrace{\frac{\partial}{\partial t}\left[\frac{\varepsilon_0 E^2}{2} + \underbrace{\frac{\mu_0 H^2}{2}}_{B^2/2\mu_0}\right]} + \vec{E}.\frac{\partial \vec{P}}{\partial t}$$

$$= \frac{\partial}{\partial t} W_e + \frac{\partial W_m}{\partial t}$$

W_e : énergie du chp EM

4) $$\frac{\partial W_m}{\partial t} = \frac{1}{2} N m \left[\frac{\partial}{\partial t}\dot{\rho}^2 + \Omega_0^2 \frac{\partial}{\partial t}\rho^2\right]$$

$\vec{P} = Nq\vec{\rho}$; $\dfrac{\partial \vec{P}}{\partial t} = Nq\dfrac{\partial \vec{\rho}}{\partial t}$

$$\frac{\partial W_m}{\partial t} = \left(\vec{E}.\frac{\partial \vec{P}}{\partial t}\right) = \frac{m}{q}\left(\ddot{\vec{\rho}} + \Omega_0^2 \vec{\rho}\right).\frac{\partial}{\partial t}(Nq\vec{\rho})$$

$$= mN\left(\ddot{\rho}\dot{\rho} + \Omega_0^2 \rho\dot{\rho}\right)$$

$$= mN\left(\frac{1}{2}\frac{\partial \dot{\rho}^2}{\partial t} + \frac{\Omega_0^2}{2}\frac{\partial}{\partial t}\rho^2\right)$$

$$\Rightarrow \frac{\partial W_m}{\partial t} = \frac{1}{2} N m \left[\frac{\partial}{\partial t}\vec{\dot{\rho}}^{\,2} + \Omega_0^2 \frac{\partial}{\partial t}\vec{\rho}^{\,2}\right]$$ cqfd.

5) $\vec{S} = \vec{E} \wedge \vec{H} = \vec{E} \wedge \frac{\vec{B}}{\mu_0} = \vec{E} \wedge \frac{1}{\mu_0}\left(\frac{\vec{k} \wedge \vec{E}}{\omega}\right) = \frac{kc\,E^2\vec{u}}{\mu_0 \omega c} = \frac{n\vec{E}^2\vec{u}}{\mu_0 c}$

n

$$\langle \vec{S} \rangle = \frac{n}{\mu_0 c} \langle \vec{E}^2 \rangle$$

6) $\langle W_m \rangle = \frac{1}{2}\varepsilon_0 \left[n^2 - 1 + \omega \frac{d(n^2-1)}{d\omega}\right] \langle \vec{E}^2 \rangle \longrightarrow$ un peu de calcul... mon demandé

$$\langle W_e \rangle = \left\langle \frac{1}{2}(\varepsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2) \right\rangle$$

$$\vec{B} = \mu_0 \vec{H} = \frac{\vec{k} \wedge \vec{E}}{\omega}$$

$$\frac{1}{2}\mu_0 \vec{H}^2 = \frac{1}{2}\mu_0 \frac{1}{\mu_0^2}\left(\frac{\vec{k} \wedge \vec{E}}{\omega}\right)^2 \qquad \text{avec } k = \frac{n\omega}{c}$$

$$= \frac{1}{2\mu_0}\frac{k^2 E^2}{\omega^2} = \frac{1}{2\mu_0}\Big(\frac{n^2\omega^2}{\underbrace{c^2}_{=1/\varepsilon_0}}\Big)\frac{\vec{E}^2}{\omega^2} = \frac{1}{2}\varepsilon_0 n^2 \vec{E}^2$$

donc $\langle W_e \rangle = \frac{1}{2}\varepsilon_0 \langle \vec{E}^2 \rangle + \frac{1}{2}\varepsilon_0 n^2 \langle \vec{E}^2 \rangle = \frac{1}{2}(n^2+1)\varepsilon_0 \langle \vec{E}^2 \rangle$

7) $\langle W \rangle = \langle W_e \rangle + \langle W_m \rangle = \frac{1}{2}(n^2 \cancel{+1})\varepsilon_0 \langle \vec{E}^2 \rangle + \frac{1}{2}(n^2 \cancel{-1})\varepsilon_0 \langle \vec{E}^2 \rangle$
$+ \frac{1}{2}\varepsilon_0 \omega \frac{d(n^2-1)}{d\omega} \langle \vec{E}^2 \rangle$

$$= \cancel{\frac{1}{2}} 2n^2 \varepsilon_0 \langle \vec{E}^2 \rangle + \frac{1}{2}\varepsilon_0 \omega \frac{d(n^2 \cancel{-1})}{d\omega} \langle \vec{E}^2 \rangle$$

$$= \frac{1}{2}\varepsilon_0 \left(2n^2 + \omega \frac{dn^2}{d\omega}\right) \langle \vec{E}^2 \rangle = n\varepsilon_0 \frac{d(n\omega)}{d\omega} \langle \vec{E}^2 \rangle$$

Rapport $\frac{\langle S \rangle}{\langle W \rangle} = \frac{\cancel{n}\, \cancel{\langle \vec{E}^2 \rangle}\, d\omega}{\mu_0 c\, \cancel{n}\varepsilon_0 \underbrace{d(n\omega)}_{= dk/c} \cancel{\langle \vec{E}^2 \rangle}} = \frac{d\omega}{dk} = v_g$ vitesse de groupe.