MARS 2007

Question de cours :

① $\vec{P}$: $\vec{P}$ = densité volumique de moment dipolaire -

α : polarisabilité : " tendance d'une entité (atome, ion, molécule) microscopique à se polariser sous l'action d'un champ électrique -

(2)

$$\vec{p} = \alpha \varepsilon_0 \vec{E_e} \qquad \alpha \text{ en } m^{-3}$$

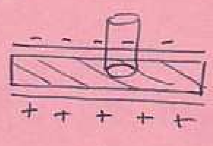
② Polarisabilité Electronique
Atomique
ionique
orientation.

(2)

Condensateur à diélectrique.

(1) -1- $E_{ext} = 0 \quad D_{ex} = \varepsilon_0 E_{ex} = 0$

-2- $\oint \vec{D}\,\vec{n}\, dS = Q$



$-DS = -\sigma S$

$\boxed{\vec{D} = \sigma \vec{e_z}}$

(2)

$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

(1) $\boxed{\vec{E} = \dfrac{\sigma \vec{e_z}}{\varepsilon_0 \varepsilon_r}}$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

(1) $\vec{P} = \sigma \vec{e_z} - \dfrac{\sigma}{\varepsilon_r} \vec{e_z} = \boxed{\sigma \left(\dfrac{\varepsilon_r - 1}{\varepsilon_r}\right) \vec{e_z}}$

-3-

(1) $\boxed{\sigma = \vec{P} \cdot \vec{n}_{ex}}$

(2) $\vec{E}_m = + -\dfrac{\sigma_s}{\varepsilon_0} \vec{e_z} = \boxed{-\dfrac{P}{\varepsilon_0} \vec{e_z} = \vec{E}_m}$

$$\vec{E}_T = \vec{E}_{cond.} + \vec{E}_m = \frac{\sigma}{\varepsilon_0} \vec{e_z} - \frac{P}{\varepsilon_0} \vec{e_z}$$

(1) $= \left(\dfrac{\sigma}{\varepsilon_0} - \dfrac{\sigma (\varepsilon_r - 1)}{\varepsilon_r \varepsilon_0}\right) \vec{e_z} = \dfrac{\varepsilon_r \sigma - \sigma \varepsilon_0 + \sigma}{\varepsilon_0 \varepsilon_r} \vec{e_z}$

$$\boxed{\vec{E}_T = \frac{\sigma}{\varepsilon_0 \varepsilon_r} \vec{e_z}}$$

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sphère unif. polarisée :

-1- $V_m(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0} \iiint \dfrac{\vec{P}\cdot(\vec{r}-\vec{r}\,')}{\|\vec{r}-\vec{r}\,'\|^3}\, d\tau$

$= \dfrac{1}{4\pi\varepsilon_0}\vec{P}\cdot \iiint \dfrac{\vec{r}-\vec{r}\,'}{\|\vec{r}-\vec{r}\,'\|^3}\, d\tau$

$= \dfrac{\vec{P}}{\rho_0}\cdot \dfrac{1}{4\pi\varepsilon_0} \iiint \rho_0 \dfrac{\vec{r}-\vec{r}\,'}{\|\vec{r}-\vec{r}\,'\|^3}\, d\tau$

(2) $\boxed{V_m(\vec{r}) = \dfrac{\vec{P}\,\vec{E}^*(\vec{r})}{\rho_0}}$

-2- Théorème de Gauss dans le vide :

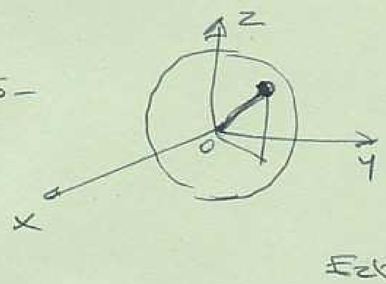
(2) $4\pi r^2 E^*(\vec{r}) = \dfrac{\frac{4}{3}\pi r^3 \rho_0}{\varepsilon_0}$ $\boxed{\vec{E}^*(\vec{r}) = \dfrac{\rho_0 \vec{r}}{3\varepsilon_0}}$

(1) -3- $V_m(\vec{r}) = \dfrac{P\vec{e_z}\cdot \rho_0 r \vec{e_r}}{3\rho_0\varepsilon_0} = \boxed{\dfrac{P\cos\theta\, r}{3\varepsilon_0}} = \dfrac{Pz}{3\varepsilon_0}$

(1) $\vec{E} = -\overrightarrow{\text{grad}}\, V = -\dfrac{\partial V}{\partial z}\vec{e_z} = -\dfrac{P}{3\varepsilon_0}\vec{e_z} = -\dfrac{\vec{P}}{3\varepsilon_0}$

(1) -4- $\sigma = \vec{P}\,\vec{n_{ex}} = P\vec{e_z}\cdot\vec{e_r} = \boxed{P\cos\theta = \sigma}$

-5-



$\vec{E}(0) = E_z(0)\,\vec{e_z}$ symétrie

$E_z(0) = E(0)\cos\theta = \dfrac{\sigma\, dS}{4\pi\varepsilon_0}\dfrac{-\vec{R}}{R^3}\cos\theta$

$E_z(0) = -\dfrac{P\cos^2\theta \cdot R^2\sin\theta d\theta d\varphi}{4\pi\varepsilon_0 R^2} = -P\cos^2\theta\sin\theta d\theta d\varphi$

$u = \cos\theta \quad du = \sin\theta$

(2) $E_z(0) = \dfrac{+P\cdot 2\pi}{4\pi\varepsilon_0}\displaystyle\int_1^{-1} u^2\, du = \dfrac{-P\cdot 2\pi}{4\pi\varepsilon_0}\dfrac{1}{3}\left[u^3\right]_{-1}^{1}$

$\boxed{E_z(0) = \dfrac{-P}{3\varepsilon_0}}$