Question de cours :

- Polarisation d'un gaz de molécules polaires sous faible pression.

(2) [1] · électronique · atomique · orientation

$$\boxed{\alpha = \alpha_e + \alpha_a + \alpha_{or}}$$

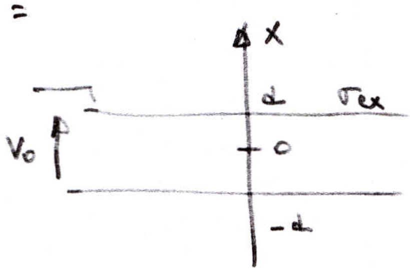
(2) [2] $\boxed{\vec P = N\vec\mu = N\alpha\varepsilon_0 \vec E_l}$

$\vec E_l = \vec E$ $\boxed{\vec P = N(\alpha_e + \alpha_a + \alpha_{or})\vec E\,\varepsilon_0}$

(1) [3] $\vec D = \varepsilon_0\varepsilon_r \vec E = \varepsilon_0 \vec E + \vec P = \varepsilon_0(1 + N(\alpha_e + \alpha_a + \alpha_{or}))\vec E$

$$\boxed{\varepsilon_r = 1 + N\alpha.}$$

Exercice 1 =



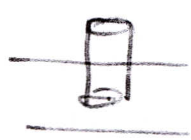
$$\varepsilon = \frac{4\varepsilon_0}{\left(\frac{x}{d}\right)^2 + 1}$$

(2) (1) [1] invariances y, z $\vec D(xyz) = \vec D(x)$

(1) sym. $\forall$ plan $\parallel x$ = symétrie $\vec D(x) = D_x(x)\vec e_x$

$\vec D$ vers les x décroissant.

[2] (1) $\oiint \vec D\,\vec n\,dS = Q_{ex}$.



(2) $\oiint \vec D\,\vec n\,dS = -D_x S = \sigma_{ex} S$ (1) $\boxed{D_x = -\sigma_{ex}}$

$\forall x,\ -d < x < d.$

(1) $\vec E = \frac{\vec D}{\varepsilon}$ $\boxed{E_x = \frac{D_x\left(\left(\frac{x}{d}\right)^2 + 1\right)}{4\varepsilon_0}x}$

C'est entre des bornes

$\vec E = -\vec{\text{grad}}\,V = -\frac{dV}{dx}\vec e_x$

[3] $V(x) = -\int_{x=-d}^{x} E(x)\,dx = -\frac{D(x)}{4\varepsilon_0}\int_{-d}^{x}\left(\left(\frac{x}{d}\right)^2 + 1\right)dx$

$= -\frac{D_x}{4\varepsilon_0}\left[\frac{1}{3}\frac{1}{d^2}x^3 + x\right]_{-d}^{x}$

$-\frac{d^3}{3d^2} - d$

$$V(x) = -\frac{D_x}{4\varepsilon_0}\left[\frac{1}{3}\frac{x^3}{d^2} + x + \frac{4}{3}d\right] = \frac{+\sigma_{ex}}{4\varepsilon_0}\left(\frac{1}{3}\frac{x^3}{d^2} + x + \frac{4}{3}d\right)$$

$x = d$: $V = V_0$ $\quad V_0 = +\frac{\sigma_{ex}}{4\varepsilon_0}\left[\frac{8}{3}d\right] \Rightarrow \frac{+\sigma_{ex}}{4\varepsilon_0} = \frac{3V_0}{8d}$ — $\sigma_{ex} = \frac{3V_0\varepsilon_0}{2d}$

2

2 4 $\quad \boxed{V(x) = \frac{V_0}{8d}\left\{\frac{x^3}{d^2} + 3x + 4d\right\}}$

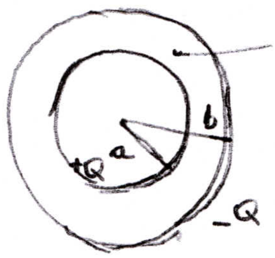
4 $\quad \vec{D}_x = \varepsilon_0\vec{E}_x + \vec{P} \qquad \vec{P} = \left(\overset{\vec{D}}{\overbrace{-\frac{1}{4}\sigma_{ex}}} + \underbrace{\frac{\sigma_{ex}\left(\left(\frac{x}{d}\right)^2+1\right)}{4}}_{-\varepsilon_0\vec{E} = \frac{-D}{4}\left(\left(\frac{x}{d}\right)^2+1\right)}\right)\vec{e_x}$

2 $\quad \boxed{\vec{P} = -\sigma_{ex}\left\{\frac{3-\left(\frac{x}{d}\right)^2}{4}\right\}\vec{e_x}} = \boxed{\frac{3\varepsilon_0V_0}{\cancel{8}\,2d}\left\{\frac{3-(x/d)^2}{4}\right\}\vec{e_x}}$

1 $\quad \operatorname{div}\vec{P} = \rho_{int} \qquad \boxed{\rho_{in} = \frac{\cancel{6}\,6\,\varepsilon_0V_0\,x}{\cancel{32}\,8\,d^3}} = \frac{Dx}{2d^2}$

$\operatorname{div}\vec{P} = \frac{\partial P}{\partial x} = -\frac{3\varepsilon_0V_0}{2d}\,x\,\frac{-2x}{4d^2}$

Exercice 2 :



$\varepsilon = \varepsilon_1 + \varepsilon_2\frac{1}{r^2}$ $\qquad \varepsilon_1 \rangle \varepsilon_2$

1 | $\operatorname{div}\vec{D} = \rho_{ex}$

2,05 | $\oiint \vec{D}\,\vec{n}\,dS = Q_{ex}$

$r < a \qquad D.4\pi r^2 = 0 \qquad D = 0$

2 $\qquad$ 1 $\qquad a < r < b \qquad D\,4\pi r^2 = Q \qquad D = \frac{Q}{4\pi r^2}$

$r > b \qquad D\,4\pi r^2 = 0 \qquad D = 0$

---

2 $\quad$ 2 $\quad$ 2 $\quad E \;\; a<r<b \qquad E = \frac{Q}{4\pi \cancel{r^2}}\frac{1}{\left(\varepsilon_1 r^2 + \frac{\varepsilon_2}{\cancel{r^2}}\right)} = \boxed{\frac{Q}{4\pi}\frac{1}{\varepsilon_2 + r^2\varepsilon_1}}$

2 $\quad$ 3 $\quad$ 1 $\quad -\int_a^b E\,dr = -\int_a^b \frac{Q}{4\pi}\frac{1}{\varepsilon_2\left(\frac{\varepsilon_2}{\varepsilon_2}+\frac{\varepsilon_1 r^2}{\varepsilon_2}\right)}dr = -\int_{\frac{\varepsilon_1^{1/2}}{\varepsilon_2}a}^{\frac{\varepsilon_1^{1/2}}{\varepsilon_1}b} \frac{Q}{4\pi\varepsilon_2}\frac{1}{1+x^2}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2}dx$

$$V_a - V_b = \frac{Q}{4\pi\sqrt{\varepsilon_1\varepsilon_2}} \int_{x_a}^{x_b} \frac{1}{1+x^2}\,dx = \frac{Q}{4\pi\sqrt{\varepsilon_1\varepsilon_2}} \left[\tan^{-1} x\right]_{x_a}^{x_b}$$

$$V_a - V_b = \frac{Q}{4\pi\sqrt{\varepsilon_1\varepsilon_2}} \cdot \cancel{\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2}} \left(\tan^{-1} x_a - \tan^{-1} x_b\right) \quad !$$

1) $\boxed{4}$ $Q = CV$

$$\boxed{C = 4\pi\sqrt{\varepsilon_1\varepsilon_2}\,\cancel{\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{1/2}} \left(\tan^{-1} x_a - \tan^{-1} x_b\right)^{-1}}$$