I Théorème de Gauss :

-1- $\oiint \vec{D}\,\vec{n}\,ds = Q_{int}.$

-2- $D(\rho)\cdot 2\pi\rho h = \sigma\, 2\pi a h \quad a<\rho<b \quad \boxed{\vec{D} = \frac{\sigma a}{\rho}\vec{e_\rho}}$

-3- $\vec{D} = \varepsilon\vec{E} \quad \vec{E} = \frac{\vec{D}}{\varepsilon_0\varepsilon_r} \quad \boxed{\vec{E} = \frac{\sigma a}{\varepsilon\rho}\vec{e_\rho}}$

-4- $V_0 = \int_a^b \vec{E}\,\vec{d\rho} = \int_a^b E(\rho)\,d\rho = \frac{\sigma a}{\varepsilon}\ln\frac{b}{a}$

$$\boxed{\sigma = \frac{\varepsilon V_0}{a\ln\frac{b}{a}}}$$

-5- $v_\varphi = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r}} = 2\,10^8\,m/s. \quad n = \frac{3}{2} \quad \rightarrow \varepsilon_r = 2{,}25$

II Polarisation ionique :

-1- $P = \sum_{i,j} q_i\vec{r_i} - q_i\vec{r_j} = \vec{0} \quad \vec{P} = \vec{0}$

-2- $\vec{P} = ne\vec{u} - ne\vec{v}$

suivant $\vec{e_z}$: $\boxed{P = neu - nev = ne(u-v)}$

-3- $e\vec{E} - m\omega_0^2\vec{u} = \vec{0} \quad \boxed{u = \frac{eE}{m\omega_0^2} = -v}$

$-e\vec{E} - m\omega_0^2\vec{v} = \vec{0}$

-a- $P = \frac{2e^2nE}{m\omega_0^2} = \varepsilon_0\chi E \quad \boxed{\chi = \frac{2ne^2}{m\varepsilon_0\omega_0^2}}$

-b- $\varepsilon_r = 1+\chi = 1 + \frac{2\cdot 3{,}2\,10^{28}\cdot(1{,}6\,10^{-19})^2}{\frac{37\,10^{-3}}{6.02\,10^{23}}\cdot 8{,}85\,10^{-12}\cdot(2\pi\,5\,10^{12})^2} = 4{,}05\,(?)$

-4- $e\underline{E_0} = -m\beta j\omega\underline{u_0} + m\omega_0^2\underline{u_0} - \omega^2\underline{u_0}$

$\underline{u_0} = \frac{eE_0/m}{\omega_0^2-\omega^2-j\beta\omega} = -\underline{v_0}$

$\underline{P} = \frac{2ne^2E_0/m}{\omega_0^2-\omega^2-j\beta\omega} \quad \underline{\chi} = \frac{2ne^2}{m\varepsilon_0[(\omega_0^2-\omega^2)-j\omega\beta]} = \frac{\chi(0)\,\omega_0^2}{\omega_0^2-\omega^2-j\omega\beta}$