Oscillateur harmonique

1) • $U(k)$ unitaire $\quad U(k) = e^{ikX}$ $\quad$ k réel

$U^+(k) = e^{-ikX}$

$$UU^+ = e^{ikX}e^{-ikX} = e^{ikX-ikX}\underbrace{e^{\frac{1}{2}[ikX,-ikX]}}_{=1} = e^0 = 1 \Rightarrow U(k) \text{ unitaire}$$

• $\sum_{n'} |\langle \varphi_n | U(k) | \varphi_{n'} \rangle|^2 = 1$ $\qquad$ relation de fermeture

$$1 = \langle \varphi_n | \varphi_n \rangle = \langle \varphi_n | UU^+ | \varphi_n \rangle = \langle \varphi_n | U \Big(\underbrace{\sum_{n'} |\varphi_{n'}\rangle\langle\varphi_{n'}|}\Big) U^+ | \varphi_n \rangle$$

$$= \sum_{n'} \langle \varphi_n | U | \varphi_{n'} \rangle \langle \varphi_{n'} | U^+ | \varphi_n \rangle = \sum_{n'} \langle \varphi_n | U | \varphi_{n'} \rangle \langle \varphi_n | U | \varphi_{n'} \rangle^*$$

$$= \sum_{n'} |\langle \varphi_n | U | \varphi_{n'} \rangle|^2$$

2) $X' = \sqrt{\frac{m\omega}{\hbar}}\, X \qquad a = \frac{X' + iP'}{\sqrt{2}} \Rightarrow X' = \frac{a^+ + a}{\sqrt{2}} \Rightarrow X = \sqrt{\frac{\hbar}{2m\omega}}(a^+ + a)$

$P' = \frac{P}{\sqrt{m\hbar\omega}} \qquad a^+ = \frac{X' - iP'}{\sqrt{2}}$

$$\bullet\ U(k) = e^{ik\sqrt{\frac{\hbar}{2m\omega}}(a^+ + a)} = e^{ik\sqrt{\frac{\hbar}{2m\omega}}a^+} e^{ik\sqrt{\frac{\hbar}{2m\omega}}a} e^{\frac{1}{2}\left(ik\sqrt{\frac{\hbar}{2m\omega}}\right)^2 [a^+, a]}$$

$$= e^{ik\sqrt{\frac{\hbar}{2m\omega}}a^+} e^{ik\sqrt{\frac{\hbar}{2m\omega}}a} e^{-\frac{\hbar k^2}{4m\omega}} = e^{i\sqrt{\frac{\hbar^2 k^2}{2m\hbar\omega}}a^+} e^{i\sqrt{\frac{\hbar^2 k^2}{2m\hbar\omega}}a} e^{-\frac{\hbar^2 k^2}{4m\hbar\omega}}$$

$$= e^{i\sqrt{\frac{E_k}{E_\omega}}a^+} e^{i\sqrt{\frac{E_k}{E_\omega}}a} e^{-\frac{E_k}{2E_\omega}}$$

3) $e^{\alpha a}|\varphi_0\rangle = \sum_n \frac{(\alpha a)^n}{n!}|\varphi_0\rangle = |\varphi_0\rangle + \cancel{\alpha a |\varphi_0\rangle} + \cancel{\dots}$

donc $e^{\alpha a}|\varphi_0\rangle = |\varphi_0\rangle$

$$\langle \varphi_n | e^{\alpha a^+} | \varphi_0 \rangle = \langle \varphi_n | \left(\sum_{n'} \frac{(\alpha a^+)^{n'}}{n'!}\right) | \varphi_0 \rangle \quad \text{or } a^{+n'}|\varphi_0\rangle = \sqrt{n'!}\,|\varphi_{n'}\rangle$$

$$= \langle \varphi_n | \left(\sum_{n'} \frac{\alpha^{n'}\sqrt{n'!}}{n'!}\right) | \varphi_{n'} \rangle = \sum_{n'} \langle \varphi_n | \varphi_{n'} \rangle \frac{\alpha^{n'}}{\sqrt{n'!}} = \frac{\alpha^n}{\sqrt{n!}}$$

$\neq 0$ si $n = n'$

donc $\langle \varphi_n | e^{\alpha a^+} | \varphi_0 \rangle = \frac{\alpha^n}{\sqrt{n!}}$

$$\langle \varphi_0 | U(k) | \varphi_n \rangle = \langle \varphi_0 | e^{\frac{-E_k}{2E_\omega}} e^{i\sqrt{\frac{E_k}{E_\omega}}a^+} e^{i\sqrt{\frac{E_k}{E_\omega}}a} | \varphi_n \rangle$$

$$= e^{\frac{-E_k}{2E_\omega}} \langle \varphi_0 | e^{i\sqrt{\frac{E_k}{E_\omega}}a^+} e^{i\sqrt{\frac{E_k}{E_\omega}}a} | \varphi_n \rangle$$

$$= e^{\frac{-E_k}{2E_\omega}} \langle \varphi_n | \left(e^{i\sqrt{\frac{E_k}{E_\omega}}a^+} e^{i\sqrt{\frac{E_k}{E_\omega}}a} \right)^+ | \varphi_0 \rangle^*$$

$$= e^{\frac{-E_k}{2E_\omega}} \langle \varphi_n | \left(e^{-i\sqrt{\frac{E_k}{E_\omega}}a^+} e^{-i\sqrt{\frac{E_k}{E_\omega}}a} \right) | \varphi_0 \rangle^*$$

$$= e^{\frac{-E_k}{2E_\omega}} \langle \varphi_n | e^{-i\sqrt{\frac{E_k}{E_\omega}}a^+} | \varphi_0 \rangle^*$$

$$= e^{\frac{-E_k}{2E_\omega}} \left(\frac{\left(-i\sqrt{\frac{E_k}{E_\omega}}\right)^n}{\sqrt{n!}} \right)^* = e^{\frac{-E_k}{2E_\omega}} \left(\frac{\left(i\sqrt{\frac{E_k}{E_\omega}}\right)^n}{\sqrt{n!}} \right) = e^{\frac{-E_k}{2E_\omega}} \frac{i^n}{\sqrt{n!}} \left(\frac{E_k}{E_\omega}\right)^{n/2}$$

$k \to 0 \quad E_k \to 0 \quad \left.\begin{array}{l} e^{\frac{-E_k}{2E_\omega}} \to 1 \\ \left(\frac{E_k}{E_\omega}\right)^{n/2} \to 0 \end{array}\right\} \langle \varphi_0 | U(k) | \varphi_n \rangle \underset{n \neq 0}{\to} 0$

on pouvait le prédire car $k \to 0 \quad U(k) = e^{ikX} \to \mathbb{1} \Rightarrow \langle \varphi_0 | \varphi_n \rangle \underset{n\neq 0}{=} 0$

état fondamental

Atome d'hydrogène dans un champ magnétique

1) $H = \frac{\vec{u}^2}{2m} + V(\vec{r}) = \frac{1}{2m}\left[\vec{P} - e\vec{A}\right]^2 + V(\vec{r})$ avec $\vec{A} = \frac{1}{2}\vec{B} \wedge \vec{r}$

$\vec{a}.(\vec{b}\wedge\vec{c}) = \vec{b}.(\vec{c}\wedge\vec{a})$

$$= \frac{1}{2m}\left(\vec{P} - \frac{e}{2}\vec{B}\wedge\vec{r}\right)\left(\vec{P} - \frac{e}{2}\vec{B}\wedge\vec{r}\right) + V(\vec{r})$$

$$= \frac{\vec{P}^2}{2m} - \frac{e}{4m}\left[(\vec{B}\wedge\vec{r}).\vec{P} + \vec{P}.(\vec{B}\wedge\vec{r})\right] + \frac{e^2}{8m}|\vec{B}\wedge\vec{r}|^2 + V(\vec{r})$$

$$= \frac{\vec{P}^2}{2m} + V(\vec{r}) - \frac{e}{4m}\left[\vec{B}.(\vec{r}\wedge\vec{P}) + \vec{B}.(\vec{r}\wedge\vec{P})\right] + \frac{e^2}{8m}|\vec{B}\wedge\vec{r}|^2$$

$$= H_0 - \frac{e}{2m}\vec{B}.\vec{L} + \frac{e^2}{8m}|\vec{B}\wedge\vec{r}|^2$$

particule non perturbée

termes quadratiques $\Rightarrow$ négligeables devant les 2 autres

$$\simeq H_0 - \frac{e\hbar}{2m\hbar}\vec{B}.\vec{L} = H_0 - \frac{\mu_B}{\hbar}\vec{L}.\vec{B}$$

2) terme $-\frac{\mu_B}{\hbar}\vec{L}.\vec{B} = -\frac{e}{2m}\vec{L}.\vec{B} = -\vec{\mathcal{M}}.\vec{B}$ couplage entre m^t magnétique de H (dipolaire) et le chp magnétique

3) ψ_{nlm} fcts propres de $H = H_0 - \frac{\mu_B}{\hbar}\vec{L}.\vec{B} = H_0 - \frac{\mu_B}{\hbar} B L_z$

$$H|\psi_{nlm}\rangle = (H_0 - \frac{\mu_B B}{\hbar} L_z)|\psi_{nlm}\rangle = H_0|\psi_{nlm}\rangle - \frac{\mu_B B}{\hbar}\hat{L}_z|\psi_{nlm}\rangle$$

avec $\hat{L}_z|\psi_{nlm}\rangle = m\hbar|\psi_{nlm}\rangle$, il vient :
et $H_0|\psi_{nlm}\rangle = E^{(0)}_{nlm}|\psi_{nlm}\rangle$

$$H|\psi_{nlm}\rangle = E^{(0)}_{nlm}|\psi_{nlm}\rangle - \frac{\mu_B B}{\hbar} m\hbar|\psi_{nlm}\rangle = \underbrace{(E^{(0)}_{nlm} - m\mu_B B)}_{\text{énergie propre associée}}|\psi_{nlm}\rangle$$

≡ modif des niveaux.

4) $E_{nlm} = E^{(0)}_{nlm} - m\mu_B \frac{B}{\hbar}\hbar = E^{(0)}_{nlm} + m\hbar\omega$

5) $|\psi_{100}\rangle$ $n=1$ $l=0$ $m=0$ état 1s

$|\psi_{21m}\rangle$ $n=2$ $l=1$ $m=0\pm1$ états 2p : $|\psi_{211}\rangle, |\psi_{210}\rangle$ et $|\psi_{21-1}\rangle$

Etat $|\psi_{100}\rangle$: $E_{100} = E^{(0)}_{100} + 0\hbar\omega = -E_I$ ⇒ origine des énergies

Etats $|\psi_{21m}\rangle$: $E_{21m} = E^{(0)}_{21m} + m\hbar\omega = (\hbar\Omega + E^{(0)}_{100}) + m\hbar\omega = \hbar\Omega + m\hbar\omega - E_I$

$$|\phi_m(t=0)\rangle = \cos\alpha|\psi_{100}\rangle + \sin\alpha|\psi_{21m}\rangle$$

$$|\phi_m(t)\rangle = \cos\alpha\, e^{\frac{iE_I t}{\hbar}}|\psi_{100}\rangle + \sin\alpha\, e^{-i\frac{(\hbar\Omega + m\hbar\omega - E_I)t}{\hbar}}$$
$$= \underbrace{e^{\frac{iE_I t}{\hbar}}}_{\equiv 1}\left[\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+m\omega)t}|\psi_{21m}\rangle\right]$$

$E_I = 0$ origine des énergies

$$|\phi_m(t)\rangle = \cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+m\omega)t}|\psi_{21m}\rangle$$

6) $\begin{cases} x = r\sin\theta\cos\varphi \\ y = r\sin\theta\sin\varphi \\ z = r\cos\theta \end{cases}$

$Y_0^0(\theta,\varphi) = \frac{1}{\sqrt{4\pi}}$

$Y_1^{\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\varphi}$

$= \mp\sqrt{\frac{3}{8\pi}}\sin\theta(\cos\varphi \pm i\sin\varphi) = \mp\sqrt{\frac{3}{8\pi}}\left(\frac{x}{r} \pm i\frac{y}{r}\right)$

$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta = \sqrt{\frac{3}{4\pi}}\frac{z}{r}$

soit en écriture décondensée

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\left(\frac{x}{r}+i\frac{y}{r}\right) \quad ; \quad Y_1^{-1} = \sqrt{\frac{3}{8\pi}}\left(\frac{x}{r}-i\frac{y}{r}\right)$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\,\frac{z}{r}$$

ou $\quad x+iy = -\sqrt{\frac{8\pi}{3}}\,r\,Y_1^1 \qquad x-iy = \sqrt{\frac{8\pi}{3}}\,r\,Y_1^{-1}$

$$z = \sqrt{\frac{4\pi}{3}}\,r\,Y_1^0$$

ou encore finalement
$$\begin{cases} x = \sqrt{\frac{2\pi}{3}}\,r\left(Y_1^{-1} - Y_1^1\right) \\ y = i\sqrt{\frac{2\pi}{3}}\,r\left(Y_1^1 + Y_1^{-1}\right) \\ z = \sqrt{\frac{4\pi}{3}}\,r\,Y_1^0 \end{cases}$$

7) $\langle \psi_{211}|x|\psi_{100}\rangle$; $\langle \psi_{211}|y|\psi_{100}\rangle$; $\langle \psi_{211}|z|\psi_{100}\rangle$ pour $m=1$

$\langle \psi_{210}|x|\psi_{100}\rangle$; $\langle \psi_{210}|y|\psi_{100}\rangle$; $\langle \psi_{210}|z|\psi_{100}\rangle$ pour $m=0$

$\langle \psi_{21-1}|x|\psi_{100}\rangle$; $\langle \psi_{21-1}|y|\psi_{100}\rangle$; $\langle \psi_{21-1}|z|\psi_{100}\rangle$ pour $m=-1$

- $$\langle \psi_{211}|x|\psi_{100}\rangle = \sqrt{\frac{2\pi}{3}}\int_0^\infty R_{21}(r)R_{10}(r)\,r^3\,dr\left\{\cancel{\int Y_1^1 Y_1^{-1} Y_0^0\,d\Omega} - \int Y_1^1 Y_1^1 \underbrace{Y_0^0}_{\frac{1}{\sqrt{4\pi}}}\,d\Omega\right\} = \sqrt{\frac{2\pi}{3}}\frac{1}{\sqrt{4\pi}}\mathcal{X} = -\frac{\mathcal{X}}{\sqrt{6}}$$

$$\langle \psi_{210}|x|\psi_{100}\rangle = \sqrt{\frac{2\pi}{3}}\,\mathcal{X}\left\{\cancel{\int Y_1^0 Y_1^{-1} Y_0^0\,d\Omega} - \cancel{\int Y_1^0 Y_1^1 Y_0^0\,d\Omega}\right\} = 0$$

$$\langle \psi_{21-1}|x|\psi_{100}\rangle = \sqrt{\frac{2\pi}{3}}\,\mathcal{X}\left\{\int Y_1^{-1} Y_1^{-1} Y_0^0\,d\Omega - \cancel{\int Y_1^{-1} Y_1^1 Y_0^0\,d\Omega}\right\} = \sqrt{\frac{2\pi}{3}}\frac{1}{\sqrt{4\pi}}\mathcal{X} = \frac{\mathcal{X}}{\sqrt{6}}$$

- $$\langle \psi_{211}|y|\psi_{100}\rangle = i\sqrt{\frac{2\pi}{3}}\,\mathcal{X}\left\{\int Y_1^1 Y_1^1 Y_0^0\,d\Omega + \cancel{\int Y_1^1 Y_1^{-1} Y_0^0\,d\Omega}\right\} = i\sqrt{\frac{2\pi}{3}}\frac{1}{\sqrt{4\pi}}\mathcal{X} = \frac{i\mathcal{X}}{\sqrt{6}}$$

$$\langle \psi_{210}|y|\psi_{100}\rangle = i\sqrt{\frac{2\pi}{3}}\,\mathcal{X}\left\{\cancel{\int Y_1^0 Y_1^1 Y_0^0\,d\Omega} + \cancel{\int Y_1^0 Y_1^{-1} Y_0^0\,d\Omega}\right\} = 0$$

$$\langle \psi_{21-1}|y|\psi_{100}\rangle = i\sqrt{\frac{2\pi}{3}}\,\mathcal{X}\left\{\cancel{\int Y_1^{-1} Y_1^1 Y_0^0\,d\Omega} + \int Y_1^{-1} Y_1^{-1} Y_0^0\,d\Omega\right\} = i\sqrt{\frac{2\pi}{3}}\frac{1}{\sqrt{4\pi}}\mathcal{X} = i\frac{\mathcal{X}}{\sqrt{6}}$$

- $$\langle \psi_{211}|z|\psi_{100}\rangle = \sqrt{\frac{4\pi}{3}}\,\mathcal{X}\cancel{\int Y_1^1 Y_1^0 Y_0^0\,d\Omega} = 0$$

$$\langle \psi_{210}|z|\psi_{100}\rangle = \sqrt{\frac{4\pi}{3}}\,\mathcal{X}\int Y_1^0 Y_1^0 Y_0^0\,d\Omega = \frac{\mathcal{X}}{\sqrt{3}}$$

$$\langle \psi_{21-1}|z|\psi_{100}\rangle = \sqrt{\frac{4\pi}{3}}\,\mathcal{X}\cancel{\int Y_1^{-1} Y_1^0 Y_0^0\,d\Omega} = 0$$

$$|\phi_m(t)\rangle = \cos\alpha\,|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+m\omega)t}\,|\psi_{21m}\rangle$$

soit $|\phi_1(t)\rangle = \cos\alpha\,|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+\omega)t}\,|\psi_{211}\rangle$; $m=1$

$|\phi_0(t)\rangle = \cos\alpha\,|\psi_{100}\rangle + \sin\alpha\, e^{-i\Omega t}\,|\psi_{210}\rangle$; $m=0$

$|\phi_{-1}(t)\rangle = \cos\alpha\,|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega-\omega)t}\,|\psi_{21-1}\rangle$; $m=-1$

termes $\langle \psi_{n\ell m}|\vec{r}|\psi_{n\ell m}\rangle = 0$

• $m=1$ $\langle D_x\rangle(t) = \langle \phi_1(t)|D_x|\phi_1(t)\rangle = e\langle\phi_1(t)|x|\phi_1(t)\rangle$

$$= e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i(\Omega+\omega)t}\langle\psi_{211}|\right)x\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+\omega)t}|\psi_{211}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i(\Omega+\omega)t}\underbrace{\langle\psi_{100}|x|\psi_{211}\rangle}_{-\frac{\mathcal{X}}{\sqrt6}} + \cos\alpha\sin\alpha\, e^{i(\Omega+\omega)t}\underbrace{\langle\psi_{211}|x|\psi_{100}\rangle}_{-\frac{\mathcal{X}}{\sqrt6}}\right)$$

$$= -\frac{e\mathcal{X}}{\sqrt6}\sin(2\alpha)\cos((\Omega+\omega)t)$$

$$\langle D_y\rangle(t) = e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i(\Omega+\omega)t}\langle\psi_{211}|\right)y\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+\omega)t}|\psi_{211}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i(\Omega+\omega)t}\underbrace{\langle\psi_{100}|y|\psi_{211}\rangle}_{-\frac{i\mathcal{X}}{\sqrt6}} + \cos\alpha\sin\alpha\, e^{i(\Omega+\omega)t}\underbrace{\langle\psi_{211}|y|\psi_{100}\rangle}_{\frac{i\mathcal{X}}{\sqrt6}}\right)$$

$$= -e\frac{\mathcal{X}}{\sqrt6}\sin(2\alpha)\sin((\Omega+\omega)t)$$

$$\langle D_z\rangle(t) = e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i(\Omega+\omega)t}\langle\psi_{211}|\right)z\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega+\omega)t}|\psi_{211}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i(\Omega+\omega)t}\cancel{\langle\psi_{100}|z|\psi_{211}\rangle} + \cos\alpha\sin\alpha\, e^{i(\Omega+\omega)t}\cancel{\langle\psi_{211}|z|\psi_{100}\rangle}\right)$$

$$= 0$$

$\Rightarrow$ Pour $m=1$, $\vec{D}$ est polarisé circulairement à la pulsation $(\Omega+\omega)$ ds le plan (xOy)

• $m=0$ $\langle D_x(t)\rangle = \langle\phi_0(t)|D_x|\phi_0(t)\rangle = e\langle\phi_0(t)|x|\phi_0(t)\rangle$

$$= e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i\Omega t}\langle\psi_{210}|\right)x\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i\Omega t}|\psi_{210}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i\Omega t}\cancel{\langle\psi_{100}|x|\psi_{210}\rangle} + \cos\alpha\sin\alpha\, e^{i\Omega t}\cancel{\langle\psi_{210}|x|\psi_{100}\rangle}\right) = 0$$

$$\langle D_y(t)\rangle = \langle\phi_0(t)|D_y|\phi_0(t)\rangle = e\langle\phi_0(t)|y|\phi_0(t)\rangle$$

$$= e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i\Omega t}\langle\psi_{210}|\right)y\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i\Omega t}|\psi_{210}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i\Omega t}\cancel{\langle\psi_{100}|y|\psi_{210}\rangle} + \cos\alpha\sin\alpha\, e^{i\Omega t}\cancel{\langle\psi_{210}|y|\psi_{100}\rangle}\right) = 0$$

$$\langle D_z(t)\rangle = \langle\phi_0(t)|D_z|\phi_0(t)\rangle$$

$$= e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i\Omega t}\langle\psi_{210}|\right)z\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i\Omega t}|\psi_{210}\rangle\right)$$

$$= e\left(\cos\alpha \sin\alpha\, e^{-i\Omega t}\underbrace{\langle\psi_{100}|z|\psi_{210}\rangle}_{\mathcal{X}/\sqrt{3}} + \cos\alpha\sin\alpha\, e^{i\Omega t}\underbrace{\langle\psi_{210}|z|\psi_{100}\rangle}_{\mathcal{X}/\sqrt{3}}\right)$$

$$= e\frac{\mathcal{X}}{\sqrt{3}}\sin(2\alpha)\cos(\Omega t)$$

$\Rightarrow$ Pour $m=0$ $\vec{D}$ est polarisé rectilignement selon $\vec{Oz}$

- $m=-1$ $\quad \langle D_x(t)\rangle = \langle\phi_{-1}(t)|D_x|\phi_{-1}(t)\rangle = e\langle\phi_{-1}(t)|x|\phi_{-1}(t)\rangle$

$$= e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i(\Omega-\omega)t}\langle\psi_{21-1}|\right)x\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega-\omega)t}|\psi_{21-1}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i(\Omega-\omega)t}\underbrace{\langle\psi_{100}|x|\psi_{21-1}\rangle}_{\mathcal{X}/\sqrt{6}} + \cos\alpha\sin\alpha\, e^{i(\Omega-\omega)t}\underbrace{\langle\psi_{21-1}|x|\psi_{100}\rangle}_{\mathcal{X}/\sqrt{6}}\right)$$

$$= e\frac{\mathcal{X}}{\sqrt{6}}\sin(2\alpha)\cos((\Omega-\omega)t)$$

$$\langle D_y(t)\rangle = e\langle\phi_{-1}(t)|y|\phi_{-1}(t)\rangle$$

$$= e\left(\cos\alpha\langle\psi_{100}| + \sin\alpha\, e^{i(\Omega-\omega)t}\langle\psi_{21-1}|\right)y\left(\cos\alpha|\psi_{100}\rangle + \sin\alpha\, e^{-i(\Omega-\omega)t}|\psi_{21-1}\rangle\right)$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i(\Omega-\omega)t}\underbrace{\langle\psi_{100}|y|\psi_{21-1}\rangle}_{-i\mathcal{X}/\sqrt{6}} + \cos\alpha\sin\alpha\, e^{i(\Omega-\omega)t}\underbrace{\langle\psi_{21-1}|y|\psi_{100}\rangle}_{i\mathcal{X}/\sqrt{6}}\right)$$

$$= -e\frac{\mathcal{X}}{\sqrt{6}}\sin(2\alpha)\sin((\Omega-\omega)t)$$

$$\langle D_z(t)\rangle = e\langle\phi_{-1}(t)|z|\phi_{-1}(t)\rangle$$

$$= e\left(\cos\alpha\sin\alpha\, e^{-i(\Omega-\omega)t}\cancel{\langle\psi_{100}|z|\psi_{21-1}\rangle} + \cos\alpha\sin\alpha\, e^{i(\Omega-\omega)t}\cancel{\langle\psi_{21-1}|z|\psi_{100}\rangle}\right)$$

$$= 0$$

$\Rightarrow$ Pour $m=-1$, $\vec{D}$ est polarisé circulairement à la pulsation $(\Omega-\omega)$ ds le plan (xOy)