1) I)

a) $[t] = [\vec{L}^0] = [\vec{r}^0 \wedge \vec{p}^0] = m \times kg\frac{m}{s} = \boxed{kg\,m^2/s}$

b) $[t] = kg\frac{m^2}{s} \times \frac{c^2}{c^2} = kg \cdot c^2 \times \frac{m^2}{s}\frac{1}{m^2}s^2 = \boxed{J.s}$

2) a) $m = 0{,}1\,g$, $\ell = 5\,mm$, $T = 100\,ms$

$v = \frac{2\pi\ell}{T}$; $|\vec{L}^0| = |\vec{r}^0 \wedge \vec{p}^0| = \ell . m v$

$|\vec{L}^0| = \frac{m 2\pi \ell^2}{T} = \frac{10^{-4} \times 2\pi \times (5.10^{-3})^2}{0{,}1} = 15{,}7.10^{-8}\,kg\frac{m^2}{s}$

$[\vec{L}^0] \simeq 1{,}6.10^{-7}\,J.s$

$\boxed{\frac{|\vec{L}^0|}{t_L} = 1{,}5.10^{27}}$

b) $P = 1\,kW$; $\nu = 1\,MHz$ $\quad [P] = \left[\frac{W}{T}\right] = [W.\nu] = \frac{W}{T^2}.T = \frac{[W.\nu]}{J.s}\nu^2$

$[L] = [J.S] = \frac{[P]}{[\nu^2]} \Rightarrow L = \frac{10^3}{(10^6)^2} = \frac{10^3}{10^{12}} = 10^{-9} \Rightarrow \boxed{\frac{L}{t_L} = 9{,}5.10^{24}}$

c) $\lambda = 1215{,}4\,\text{Å} \Rightarrow \Delta E_{12} = (E_2 - E_1) = h\nu = \frac{hc}{\lambda} = \frac{6{,}62.10^{-34} \cdot 3.10^8}{1215{,}4.10^{-10}}$

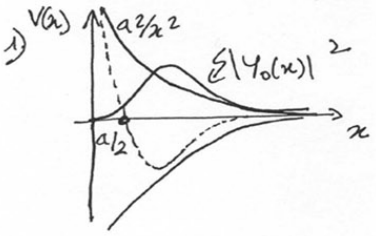
$\Delta E = 1{,}63.10^{-18}\,J = 10{,}2\,eV$. $\quad T = \frac{\lambda}{c} = 4{,}05.10^{-16}\,s$.

$[\vec{L}^0] = [\Delta E . T] = 6{,}6.10^{-34}\,J.s \Rightarrow \boxed{\frac{L}{t_L} = 6{,}28}$.

3) Cas c. car $|\vec{L}^0| \simeq t_L$

II) $V(x) = V_0\left(\frac{a^2}{x^2} - \frac{2a}{x}\right)$ $\quad V(x) = 0 \Rightarrow \frac{a^2}{x^2} = \frac{2a}{x} = 0 \quad x = \frac{a}{2}$

1)



2) $\hat{H}\varphi(x) = E\varphi(x)$; $\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\varphi(x) + V(x)\varphi(x) = E\varphi(x)}$

$E = -\mathcal{E} < 0$.

3) $\varphi_0(x) = x^\alpha e^{-\lambda x} \Rightarrow \varphi_0'(x) = \left(\alpha x^{\alpha-1} - \lambda x^\alpha\right)e^{-\lambda x}$

$\varphi_0'(x) = \left(\alpha x^{-1} - \lambda\right)\varphi_0(x) \Rightarrow \varphi_0''(x) = -\frac{\alpha}{x^2}\varphi_0 + \left(\frac{\alpha}{x} - \lambda\right)^2\varphi_0$

$\varphi_0''(x) = \left(-\frac{\alpha}{x^2} + \frac{\alpha^2}{x^2} + \lambda^2 - \frac{2\lambda\alpha}{x}\right)\varphi_0$

$-\varphi_0''(x) + \frac{2m}{\hbar^2}V_0\left(\frac{a^2}{x^2} - \frac{2a}{x}\right)\varphi_0 = \frac{2m}{\hbar^2}E\,\varphi_0$; $V_0 a^2 = \frac{\hbar^2}{m} \Rightarrow -\varphi_0''(x) + \frac{2V_0}{V_0 a^2}\left(\frac{a^2}{x^2} - \frac{2a}{x}\right)\varphi_0 = E\varphi_0$

$\Rightarrow \left[-\left(\frac{\alpha}{x^2} + \frac{\alpha^2}{x^2} + \lambda^2 - \frac{2\alpha\lambda}{x}\right) + \frac{2}{x^2} - \frac{4}{ax}\right]\varphi_0 = \frac{2mE}{\hbar^2}\varphi_0$

$\begin{cases} \frac{2\alpha\lambda}{x} = \frac{4}{ax} \Rightarrow \alpha.\lambda = \frac{2}{a} \Rightarrow \boxed{\lambda = \frac{2}{2a}} \\ +\frac{\alpha^2}{x^2} + \frac{\alpha}{x^2} + \frac{2}{x^2} = 0 \Rightarrow \alpha^2 + \alpha - 2 = 0 \Rightarrow \alpha = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \end{cases}$

$\alpha_+ = 1$ et $\alpha_- = -2$ $\quad \alpha_-$ impossible car λ_- serait alors négatif. $\Rightarrow \boxed{\alpha_+ = 1}$ et $\boxed{\lambda = \frac{2}{a}}$

$-2 + \frac{x}{a}$

$$\boxed{\varphi_0(x) = A \cdot x\, e^{-\frac{2}{a}x}}$$

4) a) $\int |\varphi_0(x)|^2 dx = 1 \Rightarrow |A|^2 \int x^2 e^{-\frac{4x}{a}} dx = 1 \Rightarrow |A|^2 \times \frac{2}{\left(\frac{4}{a}\right)^3} = 1$

$\Rightarrow |A|^2 = \frac{64}{a^3}\frac{1}{2} = \frac{32}{a^3} \Rightarrow |A| = \sqrt{\frac{32}{a^3}}$

$$\boxed{\varphi_0(x) = \sqrt{\frac{32}{a^3}}\, x\, e^{-\frac{2}{a}x}}$$

b) $\langle \hat{x} \rangle_{\varphi_0} = \int_0^{\infty} x\, |\varphi_0(x)|^2 dx = |A|^2 \int_0^{\infty} x^3 e^{-\frac{4}{a}x} dx =$

$\langle \hat{x} \rangle_{\varphi_0} = |A|^2 \times \frac{3!}{\left(\frac{4}{a}\right)^4} = \frac{32}{a^3} \cdot \frac{6}{\frac{16 \times 16}{a^4}} = \boxed{a \times \frac{3}{4}}$

c) $p = \frac{\hbar}{i}\frac{\partial}{\partial x} \Rightarrow p\varphi_0(x) = \frac{\hbar}{i} \times \left(\frac{1}{x} - \frac{2}{a}\right)\varphi_0(x)$

$\langle p \rangle_{\varphi_0} = \int \varphi_0^* p \varphi_0\, dx = \frac{\hbar}{i}\int \left(\frac{1}{x} - \frac{2}{a}\right) |\varphi_0(x)|^2 dx =$

$\frac{\hbar}{i} \int \left(\frac{1}{x} - \frac{2}{a}\right) x\, x^2 e^{-\frac{4}{a}x} dx = \frac{\hbar}{i}\left[\int x e^{-\frac{4}{a}x} dx - \frac{2}{a}\int x^2 e^{-\frac{4}{a}x} dx\right]$

$= \frac{\hbar}{i}\left[\frac{1}{\left(\frac{4}{a}\right)^2} - \frac{2}{a} \times \frac{2}{\left(\frac{4}{a}\right)^3}\right] = \frac{\hbar}{i}\left[\frac{a^2}{16} - a^2 \times \frac{4}{4^3}\right] = \boxed{0}$ logique