(1)

1/ $E<U_0 \Rightarrow$ fonction d'onde évanescente dans les barrières.

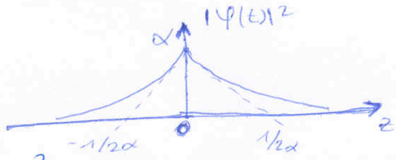
2/ $\hat{H}\varphi = E\varphi \Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\varphi + U_0\varphi = E\varphi \Rightarrow -\frac{\hbar^2}{2m}\alpha^2 + U_0 = E$

$\Rightarrow \alpha^2 = -\frac{2m(E-U_0)}{\hbar^2} \Rightarrow \boxed{\alpha = \sqrt{\frac{2m(U_0-E)}{\hbar^2}}}$ >0

3/ $\int_{-\infty}^{+\infty}|\varphi(z)|^2 dz = 1 \Rightarrow \int_{-\infty}^{0}|A|^2 e^{2\alpha z}dz + \int_0^{\infty}|A|^2 e^{-2\alpha z}dz = 1 \Rightarrow$

$\frac{1}{2\alpha}|A|^2\left[e^{2\alpha z}\right]_{-\infty}^{0} + \frac{-1}{2\alpha}|A|^2\left[e^{-2\alpha z}\right]_0^{\infty} = 1 \Rightarrow \frac{|A|^2}{2\alpha}+\frac{|A|^2}{2\alpha} = 1 \Rightarrow \boxed{|A| = \sqrt{\alpha}}$

$\boxed{\varphi(z) = \sqrt{\alpha}\, e^{-\alpha|z|}}$



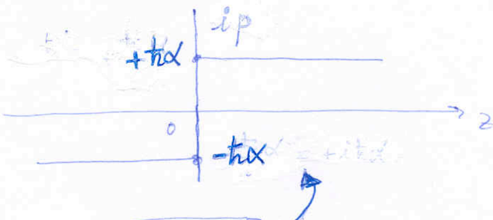
4/ $\boxed{z\varphi(z) \neq cte\ \varphi(z)}$ et $-\frac{\hbar}{i}\frac{\partial}{\partial z}\varphi(z) \overset{?}{=}$ constante $\times\,\varphi(z)\ \forall z$.

si $z<0$ $\boxed{-\frac{\hbar}{i}\frac{\partial}{\partial z}Ae^{+\alpha z}} = \boxed{-\frac{\hbar}{i}\alpha\varphi(z)}$ si $z>0$ $\boxed{-\frac{\hbar}{i}\frac{\partial}{\partial z}Ae^{-\alpha z} = +\frac{\hbar}{i}\alpha e^{-\alpha z} = \frac{\hbar}{i}\alpha\varphi(z)}$

$p \to \frac{\hbar}{i}\alpha$ si $z<0$ $\Rightarrow p \neq cte\ \forall z$

$p \to +\frac{\hbar}{i}\alpha$ si $z>0$

$\boxed{\varphi(z) \text{ n'est pas fonction propre de } \hat{z} \text{ et } \hat{p}}$

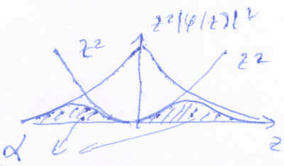


5/ $\varphi(z)$ symétrique $\frac{\partial}{\partial z}\varphi(z)$ antisymétrique $\Rightarrow \boxed{\langle p\rangle = 0}$

$\langle p^2\rangle \Rightarrow \langle\hat{H}\rangle = E \Rightarrow \langle\frac{p^2}{2m}+U_0\rangle = E \Rightarrow \langle p^2\rangle = (E-U_0)2m$

$\boxed{\langle p^2\rangle = -2m(U_0-E) = -\hbar^2\alpha^2}$ $\boxed{\langle p^2\rangle = i^2\hbar^2\alpha^2}$

$\Delta p = \sqrt{\langle p^2\rangle - \underset{0}{\langle p\rangle}^2} \Rightarrow$ $\boxed{\Delta p = i\hbar\alpha}$



6/ $z\varphi(z)$ est antisymétrique $\Rightarrow \boxed{\langle z\rangle_\varphi = 0}$

$\langle z^2\rangle = \int z^2|\varphi(z)|^2dz = \int_{-\infty}^{+\infty} z^2|A|^2 e^{-2\alpha|z|}dz = 2\cdot\int_0^{\infty} z^2|A|^2 e^{-2\alpha z}dz$; $|A|^2 = \alpha$

$\begin{matrix} u = z^2 \\ v' = e^{-2\alpha z}\end{matrix}\left|\begin{matrix} u' = 2z \\ v = -\frac{1}{2\alpha}e^{-2\alpha z}\end{matrix}\right. \Rightarrow 2\alpha\left\{\underbrace{\left[\frac{-z^2}{2\alpha}e^{-2\alpha z}\right]_0^{\infty}}_{0} + \int_0^{\infty}\frac{2ze^{-2\alpha z}}{+2\alpha}dz\right\} = \int_0^{\infty}2ze^{-2\alpha z}dz$

$\begin{matrix} u = z \\ v' = e^{-2\alpha z}\end{matrix}\left|\begin{matrix} u' = 1 \\ v = -\frac{1}{2\alpha}e^{-2\alpha z}\end{matrix}\right.$ $2\left\{\left[\frac{-z}{2\alpha}e^{-2\alpha z}\right]_0^{\infty} + \frac{1}{2\alpha}\int_0^{\infty}e^{-2\alpha z}dz\right\} = \frac{1}{\alpha}\int_0^{\infty}e^{-2\alpha z} = \left[\frac{-1}{2\alpha^2}e^{-2\alpha z}\right]_0^{\infty} = \frac{1}{2\alpha^2}$

$$\boxed{\langle z^2\rangle = \frac{1}{2\alpha^2}}$$

$$\boxed{\Delta z = \sqrt{\langle z^2\rangle - \underbrace{\langle z\rangle^2}_{0}} = \sqrt{\langle z^2\rangle} = \frac{1}{\sqrt{2}\,\alpha}}$$

4/ $$\boxed{\Delta z\,\Delta p = \frac{1}{\sqrt{2}\,\alpha} \times i\hbar\,\alpha = \frac{i\hbar}{\sqrt{2}}}$$