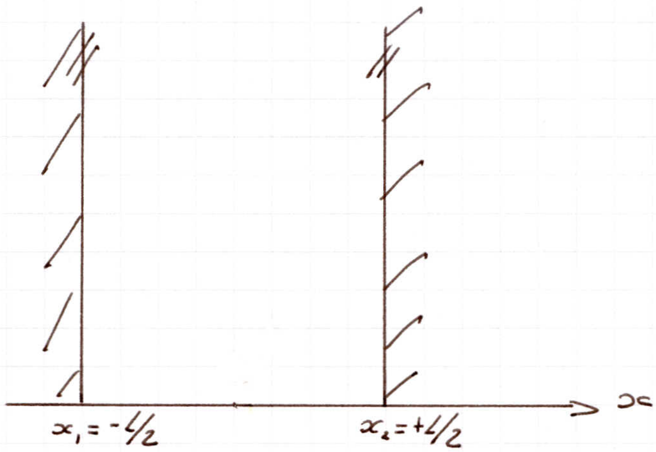
Puits de potentiel infini à une dimension



1) Eq de Schrödinger $\qquad \phi(x < -L/2) = \phi(x > L/2) = 0$

2 $\qquad$ Entre $-L/2 \leq x \leq +L/2$ $(V=0)$ $\qquad -\dfrac{\hbar^2}{2m}\dfrac{\partial^2\phi(x)}{\partial x^2} = E\,\phi(x)$

$$\frac{\partial^2\phi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\phi(x) = 0 \qquad k^2 = \frac{2mE}{\hbar^2}$$

soit $k = \sqrt{\dfrac{2mE}{\hbar^2}}$

2 $\qquad$ Expression g^{ale} de la f^{ct} d'onde : $\phi(x) = A\cos(kx) + B\sin(kx)$

2) a) Solutions paires $\qquad B = 0$ $\qquad \cos\left(k\dfrac{L}{2}\right) = 0$ $\qquad$ on a $\underbrace{\phi(-L/2) = \phi(+L/2) = 0}_{\text{continuité}}$

1,5

$$\frac{kL}{2} = \frac{\pi}{2} + n\pi \qquad n \text{ entier}$$

soit $k = (2n+1)\dfrac{\pi}{L}$

1,5 $\rightarrow$ énergie $\quad E_n = \dfrac{\hbar^2 k^2}{2m} = \dfrac{\hbar^2 (2n+1)^2 \pi^2}{2mL^2}$

b) Solutions impaires $\qquad A = 0$ $\qquad \sin\left(\dfrac{kL}{2}\right) = 0$

1,5

$$\frac{kL}{2} = n\pi \qquad n \text{ entier}$$

soit $k = \dfrac{2n\pi}{L}$

$\rightarrow$ énergie $\quad E_n = \dfrac{\hbar^2 k^2}{2m} = \dfrac{\hbar^2 (2n)^2 \pi^2}{2mL^2}$ $\qquad$ 1,5

3) Applications numériques

$$L = 10^{-10}\,m \;,\; \hbar = 1{,}055\,10^{-34}\,J.s \;,\; m_e = 9{,}1\,10^{-31}\,kg.$$

3 premiers niveaux

$n = 0$ $\begin{cases} \text{solution paire} & E_0 = 37{,}68\,eV \\ \text{solution impaire} & E_0 = 0 \end{cases}$ ① ETAT PAIR

2

$n = 1$ $\begin{cases} \text{solution paire} & E_1 = 339{,}09\,eV \\ \text{solution impaire} & E_1 = 150{,}71\,eV \end{cases}$ ③ ETAT PAIR, ② ETAT IMPAIR

4) Allure des fcts d'onde.

①

3x1