Remarques

(I) 1) Eq. de Schroedinger pour états stationnaires : $\boxed{\hat{H}\phi_0(x) = E_0\phi_0(x)}$

$$\frac{\partial\phi_0}{\partial x} = -\frac{x}{b^2}\left(\frac{1}{\pi b^2}\right)^{1/4}\exp(-x^2/2b^2) \quad (1)$$

$$\Rightarrow \frac{\partial^2\phi_0}{\partial x^2} = \left(-\frac{1}{b^2} + \frac{x^2}{b^4}\right)\left(\frac{1}{\pi b^2}\right)^{1/4}\exp(-x^2/2b^2)$$

si $\hat{H}\psi_0$ est faux mais que $\frac{\partial^2\phi_0}{\partial x^2}$ est juste : compter (2) pts
si seul $\frac{\partial\phi_0}{\partial x}$ est juste ⇒ (1) pt

$$\Rightarrow \boxed{\hat{H}\phi_0 = \frac{\hbar^2}{2mb^2}\phi_0 + \left(\frac{1}{2}m\omega^2 - \frac{\hbar^2}{2mb^4}\right)x^2\phi_0} \quad (3)$$

$$\hat{H}\phi_0 = E_0\phi_0 \Rightarrow \begin{cases} \frac{1}{2}m\omega^2 = \frac{1}{2}\frac{\hbar^2}{mb^4} \Rightarrow \boxed{b = \sqrt{\frac{\hbar^2}{m\omega}}} \quad (1) \\ E_0 = \frac{\hbar^2}{2mb^2} \end{cases}$$

2) $E_0 = \frac{\hbar^2}{2mb^2} \Longrightarrow E_0 = \frac{1}{2}\hbar\omega$ (1)

3) • $[\hbar]$ = Energie × Temps = $ML^2T^{-2} \times T = ML^2T^{-1}$

$[\omega] = T^{-1}$

$$\rightarrow [b] = \left(\frac{ML^2T^{-1}}{MT^{-1}}\right)^{1/2} = L$$

si $[b] = L$ mais pas d'explication : 1 seul pt.

• ou bien : dans $\exp\left\{-\frac{x^2}{2b^2}\right\}$ il faut que $\frac{-x^2}{2b^2}$ soit sans dimension $\Rightarrow \boxed{[b] = L}$ (2)

4) $$\frac{\partial\phi_1}{\partial x} = \left(\frac{4}{\pi b^6}\right)^{1/4}\exp(-x^2/2b^2) - \frac{x}{b^2}\left(\frac{4}{\pi b^6}\right)^{1/4}\exp\left(-\frac{x^2}{2b^2}\right)$$

$$\rightarrow \frac{\partial^2\phi_1}{\partial x^2} = -\frac{x}{b^2}\left(\frac{4}{\pi b^6}\right)^{1/4}\exp(-x^2/2b^2) - \frac{2x}{b^2}\left(\frac{4}{\pi b^6}\right)^{1/4}\exp\left(-\frac{x^2}{2b^2}\right) + \frac{x^2}{b^2}\left(\frac{4}{\pi b^6}\right)^{1/4}\frac{x}{b^2}\exp(-x^2/2b^2) \quad (3)$$

on remarque qu'avec I-1.
s'ils font d'entrée le calcul avec $b = \sqrt{\frac{\hbar^2}{m\omega}}$ et obtiennent $\hat{H}\psi_1 = E_1\psi_1$ compter 4 pts.

$$\rightarrow \boxed{\hat{H}\phi_1 = \frac{3}{2}\frac{\hbar^2}{mb^2}\phi_1(x) + \left(\frac{1}{2}m\omega^2 - \frac{\hbar^2}{2mb^4}\right)x^2\phi_1(x)} \quad (1)$$

$\hat{H}\phi_1 = E_1\phi_1 \rightarrow \frac{1}{2}m\omega^2 = \frac{\hbar^2}{3mb^4} \rightarrow$ $\boxed{\text{on retrouve } b = \sqrt{\frac{\hbar^2}{m\omega}}}$

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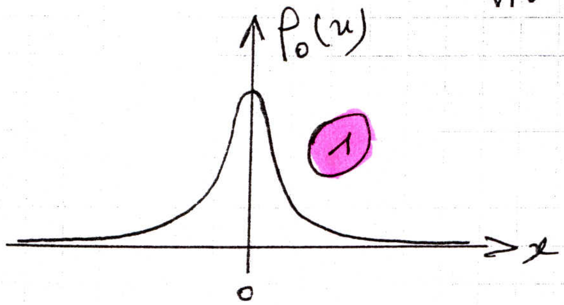
5] $E_1 = \frac{3}{2}\frac{\hbar^2}{mb^2} \Rightarrow E_1 = \frac{3}{2}\hbar\omega$ 1

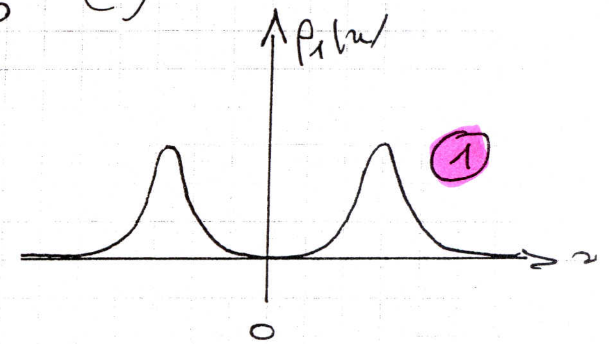
6] A.N. $b = 5.10^{-10}\,m$ 2

7]

$$\rho_0(x) = |\phi_0(x)|^2 = \frac{1}{\sqrt{\pi}\,b}\exp(-x^2/b^2)$$ 1

$$\rho_1(x) = |\phi_1(x)|^2 = \frac{2}{\sqrt{\pi}\,b}\left(\frac{x}{b}\right)^2\exp(-x^2/b^2)$$ 1





(II) 1] • $\hat{H}\Psi(\vec{r},t) = \frac{1}{\sqrt{2}}\{\hat{H}\Psi_0(\vec{r},t) + \hat{H}\Psi_1(\vec{r},t)\} = \frac{1}{\sqrt{2}}\{E_0\Psi_0(\vec{r},t) + E_1\Psi_1(\vec{r},t)\}$ 1

• $i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = \frac{i\hbar}{\sqrt{2}}\left\{\frac{\partial\Psi_0(\vec{r},t)}{\partial t} + \frac{\partial\Psi_1(\vec{r},t)}{\partial t}\right\} = \frac{1}{\sqrt{2}}\{E_0\Psi_0(x,t) + E_1\Psi_1(x,t)\}$ 1

• $\Rightarrow \hat{H}\Psi(\vec{r},t) = i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}$ vérifié 1

2] On n'a pas $\hat{H}\Psi(\vec{r},t) = E\Psi(\vec{r},t)$ 1

$\Rightarrow \Psi(\vec{r},t)$ n'est pas 1 état stationnaire

donner ce pt s'ils écrivent $\int_{-\infty}^{\infty}|\Psi(x,t)|^2 dx = \dots$

3] $\int_{-\infty}^{\infty}|\Psi(x,t)|^2 dx = \int_{-\infty}^{\infty}\Psi(x,t)\Psi^*(x,t)\,dx$ 1

$= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2}}[\Psi_0 + \Psi_1]\times\frac{1}{\sqrt{2}}[\Psi_0^* + \Psi_1^*]\,dx$

$= \frac{1}{2}\left\{\int_{-\infty}^{\infty}|\Psi_0(x,t)|^2 dx + \int_{-\infty}^{\infty}|\Psi_1(x,t)|^2 dx + \int_{-\infty}^{\infty}2\mathrm{Re}\{\Psi_0(x,t)\Psi_1^*(x,t)\}dx\right\}$

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$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = \frac{1}{2}\Big\{ \underbrace{\int_{-\infty}^{\infty} |\phi_0(x)|^2 dx}_{=1 \text{ car } \phi_0 \text{ normée}} \ (0{,}5) \ (0{,}5) + \underbrace{\int_{-\infty}^{\infty} |\phi_1(x)|^2 dx}_{=1 \text{ car } \phi_1 \text{ normée}} \ (0{,}5) \ (0{,}5)$$

$$+ 2\cos\left(\frac{E_1-E_0}{\hbar}t\right) \underbrace{\int_{-\infty}^{\infty} \phi_0(x)\phi_1^*(x)\, dx}_{=0 \text{ (intégrale d'1 fct impaire)}} \ (0{,}5) \Big\} \ (0{,}5)$$

$$\Rightarrow \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1 \ (1) \longrightarrow \Psi(x,t) \text{ est normée}$$

4] $\Rightarrow \rho(x,t) = \Psi(x,t).\Psi^*(x,t)$ (1)

$$= |\phi_0(x)t)|^2 + |\phi_1(x)|^2 + 2\cos\left[\frac{E_1-E_0}{\hbar}t\right]\phi_0(x).\phi_1^*(x) \ (1)$$

$$\rightarrow \rho(x,t) = \frac{1}{\sqrt{\pi}\, b}\exp\left(-\frac{x^2}{b^2}\right) + \frac{2}{\sqrt{\pi}\, b^3} x^2 \exp\left(-\frac{x^2}{b^2}\right)$$

$$+ 2\frac{1}{\sqrt{\pi}\, b^2} x \exp(-x^2/b^2).\cos\left[\frac{E_1-E_0}{\hbar}t\right]$$

$$\longrightarrow \begin{cases} f_1(x) = \dfrac{1}{\sqrt{\pi}\, b}\left[1 + 2\left(\dfrac{x}{b}\right)^2\right]\exp(-x^2/b^2) \ (1) \\ f_2(x) = \dfrac{2}{\sqrt{\pi}.b}\left(\dfrac{x}{b}\right)\exp(-x^2/b^2) \ (1) \\ \omega_{01} = \dfrac{E_1-E_0}{\hbar} = \dfrac{1}{\hbar}\left[\dfrac{3}{2}\hbar\omega - \dfrac{1}{2}\hbar\omega\right] = \omega \ (1) \end{cases}$$

5] $\langle x(t)\rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\, x\, \Psi(x,t)\, dx$

(1)

$$= \int_{-\infty}^{\infty} \rho(x,t).\, x\, dx$$

$$\searrow = \underbrace{\int_{-\infty}^{\infty} x\, f_1(x)\, dx}_{\text{impaire} \rightarrow \text{intégrale}=0} + \frac{2}{\sqrt{\pi}\, b}\cos(\omega t)\int_{-\infty}^{\infty} x.\left(\frac{x}{b}\right)\exp\left(-\frac{x^2}{b^2}\right)dx \ (2)$$

Si la somme de 2 intégrale est donnée mais pas le fait que la 1er est nulle → 1.5 pt

$$\langle x(t)\rangle = \frac{2}{\sqrt{\pi}}\, b \cos(\omega t) \underbrace{\int_{-\infty}^{\infty} y^2 e^{-y^2}\, dy}_{\frac{\sqrt{\pi}}{2}}$$ 1

$$\Rightarrow \langle x(t)\rangle = b \cos(\omega t)$$ 1

b)

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