1) - $\left.\begin{array}{l}\vec{\Omega}(R_P \mid R_0) = \Omega\,\vec{e}_z \\ \vec{\Omega}(R_D \mid R_P) = \omega\,\vec{e}_z\end{array}\right\} \Rightarrow \vec{\Omega}(R_D \mid R_0) = \vec{\Omega}(R_D \mid R_P) + \Omega(R_P \mid R_0) = (\Omega + \omega)\,\vec{e}_z$

2) $\vec{v}(M / R_P) = \left.\dfrac{d\,\vec{OM}}{dt}\right]_{R_P} = \left.\dfrac{d}{dt}\left(\vec{OC} + \vec{CM}\right)\right]_{R_P} = \left.\dfrac{d}{dt}\Big(\underset{\uparrow\ \text{fixedsR}_P}{L\vec{e}_{x'}} + \underset{\text{cte}}{r}\,\vec{e}_1\Big)\right]_{R_P} = r\left.\dfrac{d\vec{e}_1}{dt}\right]_{R_P} = r\omega\,\vec{e}_2$

$\vec{v}_e(M, R_P / R_0) = \underset{= \vec{0}}{\vec{v}(O \in R_P \mid R_0)} + \vec{\Omega}(R_P \mid R_0) \times \vec{OM} = \Omega\,\vec{e}_z \times (L\,\vec{e}_{x'} + r\,\vec{e}_1) = \Omega(L\,\vec{e}_{y'} + r$

$\vec{v}(M / R_0) = \left.\dfrac{d}{dt}\,\vec{OM}\right]_{R_0} = L\left[\dfrac{d\vec{e}_{x'}}{dt}\right]_{R_0} + r\left[\dfrac{d\vec{e}_1}{dt}\right]_{R_0} = L\,\Omega\,\vec{e}_{y'} + r\,(\Omega + \omega)\,\vec{e}_2$

$\Rightarrow \vec{v}(M \mid R_0) = \vec{v}(M \mid R_P) + \vec{v}_e(M, R_P \mid R_0)$ (CQFD)

3) $\vec{a}(M / R_P) = \left.\dfrac{d\,\vec{v}(M \mid R_P)}{dt}\right]_{R_P} = r\omega\left[\dfrac{d\vec{e}_2}{dt}\right]_{R_P} = -R\omega^2\,\vec{e}_1$

$\vec{a}_C(M, R_P \mid R_0) = 2\,\vec{\Omega}(R_P \mid R_0) \times \vec{v}(M / R_P) = 2\,\Omega\,\vec{e}_z \times r\omega\,\vec{e}_2 = -2r\,\Omega\omega\,\vec{e}_1$

$\vec{a}_e(M, R_P \mid R_0) = \vec{a}(O_P \mid R_0) + \vec{\Omega}(R_P \mid R_0) \times \left[\vec{\Omega}(R_P \mid R_0) \times \vec{O_P M}\right] + \underbrace{\left(\dfrac{d\vec{\Omega}(R_P \mid R_0)}{dt}\right]_{R_0}}_{= \vec{0}} \times \vec{O_P M}$

$= \vec{0} + \Omega\,\vec{e}_z \times \left[\Omega\,\vec{e}_z \times (L\,\vec{e}_{x'} + r\,\vec{e}_1)\right]$

$= \Omega\,\vec{e}_z \times \left(\Omega L\,\vec{e}_{y'} + \Omega r\,\vec{e}_2\right) = -\Omega^2\left(L\,\vec{e}_{x'} + r\,\vec{e}_1\right)$

$\vec{a}(M / R_0) = \left[\dfrac{d}{dt}\,\vec{v}(M \mid R_0)\right]_{R_0} = \left[\dfrac{d}{dt}\left[L\Omega\,\vec{e}_{y'} + r(\Omega + \omega)\,\vec{e}_2\right]\right]_{R_0} = -L\Omega^2\,\vec{e}_{x'} - r(\Omega + \omega)^2\,\vec{e}_1$

$\vec{a}(M / R_P) + \vec{a}_C(M, R_P / R_0) + \vec{a}_e(M, R_P / R_0) = \underbrace{\left(-R\omega^2 - 2r\,\Omega\omega - r\Omega^2\right)}_{-r(\Omega + \omega)^2}\vec{e}_1 - L\Omega^2\,\vec{e}_{x'}$

$= \vec{a}(M \mid R_0)$ (CQFD)

1- $R_0 = (T, \underbrace{\vec{e}_{x_0}, \vec{e}_{y_0}, \vec{e}_{z_0}})$ axes fixes (3 étoiles) / Képler ou Copernic

2- Th. de Gauss : $\oiint_{(S)} \vec{g}\cdot\vec{n}\, dS = -4\pi G M_{\text{int à }(S)}$ et $\vec{g}(r) = -g(r)\,\vec{e}_r$

$(S) \rightarrow$ sphère de centre T et rayon r

$\Rightarrow -g(r)\cdot 4\pi r^2 = -4\pi G M \iff g(r) = +\dfrac{GM}{r^2} = +\dfrac{GM}{(R+z)^2}$

$g_0 = g(R) = +\dfrac{GM}{R^2} \Rightarrow GM = g_0 R^2$

$\Rightarrow g(r) = g_0 \dfrac{R^2}{(R+z)^2}$

3- $\Sigma \vec{F} = m\vec{a} = -m\dfrac{v^2}{r}\vec{e}_r + m\dfrac{dV}{dt}\vec{e}_\theta$

$\iff -\dfrac{G\cancel{m}M}{r^{\cancel{2}}}\vec{e}_r = -\dfrac{\cancel{m}v^2}{\cancel{r}}\vec{e}_r$ et $\dfrac{dV}{dt} = 0 \Rightarrow V = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{g_0 R^2}{r}}$

4- $T = \dfrac{2\pi r}{V} = \dfrac{2\pi}{R}\sqrt{\dfrac{r^3}{g_0}}$

5- $E_k = \dfrac{1}{2} m v^2 = \dfrac{1}{2}\dfrac{GmM}{r} = \dfrac{1}{2} m g_0 \dfrac{R^2}{r}$, $E_p = -\dfrac{GmM}{r} = -m g_0 \dfrac{R^2}{r} = -2E_k$

6- $E_m = E_k + E_p = -\dfrac{1}{2} m g_0 \dfrac{R^2}{r} < 0 \Rightarrow$ état lié.

7- Géostationnaire s'il est tjs à la verticale du m̂ pt de la Terre.

8- Mvt à force centrale $\Rightarrow$ trajectoire ds un plan $\ni$ le centre de force, et $\perp \vec{\Omega}$

$\Rightarrow$ mvt ds le plan équatorial. (géost $\Rightarrow T = 24h \Rightarrow h = 35.930$ km)

9- NON - Paris $\notin$ plan équatorial.